Package 'uniformly'

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Type Package

Title Uniform Sampling

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Description Uniform sampling on various geometric shapes, such as spheres, ellipsoids, simplices.

License GPL-3

URL https://github.com/stla/uniformly

BugReports https://github.com/stla/uniformly/issues

Imports abind, pgnorm, rgl, stats

Suggests geometry, knitr, misc3d, rmarkdown, scatterplot3d

VignetteBuilder knitr

Encoding UTF-8

RoxygenNote 7.2.3

NeedsCompilation no

Repository CRAN

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Contents

`makeHexahedron` *Make hexahedron*

Description

Make a hexahedron for usage in `runif_in_hexahedron` and other functions.

Usage

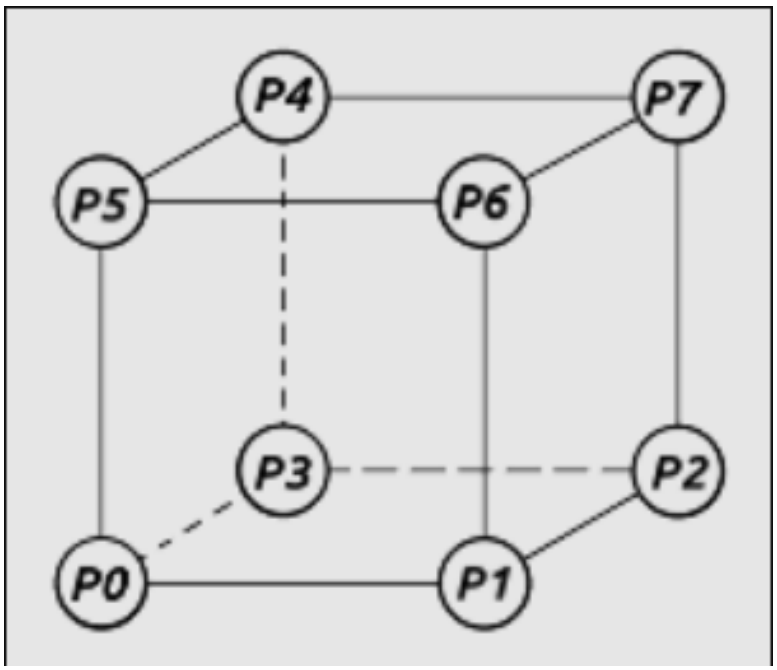
```
makeHexahedron(p0, p1, p2, p3, p4, p5, p6, p7)
```

Arguments

`p0`, `p1`, `p2`, `p3`, `p4`, `p5`, `p6`, `p7`
: the eight vertices of the hexahedron, as in the figure shown below

Details

A hexahedron is a polyhedron having six quad faces. Its eight vertices must be placed as in the figure below.



Value

A matrix with eight columns, the vertices.

See Also

The function `plotHexahedron` is useful to check the hexahedron.

Examples

```
library(uniformly)
# a non-convex hexahedron
hexahedron <- makeHexahedron(
  p0 = c(1.5, 1.5, 0),
  p1 = c(2, 0, 0),
  p2 = c(2, 2, 0),
  p3 = c(0, 2, 0),
  p4 = c(0, 2, 2),
  p5 = c(0, 0, 2),
  p6 = c(2, 0, 2),
  p7 = c(2, 2, 2)
)
plotHexahedron(hexahedron)
```

plotHexahedron *Plot hexahedron*

Description

Plot a hexahedron with **rgl**.

Usage

```
plotHexahedron(hexahedron, alpha = 1)
```

Arguments

hexahedron	a hexahedron given by a 3 times 8 matrix; see makeHexahedron
alpha	opacity, a number between 0 and 1

Value

No returned value, called for plotting.

Examples

```
library(uniformly)
hexahedron <- makeHexahedron(
  p0 = c(0, 0, 0),
  p1 = c(2, 0, 0),
  p2 = c(2, 2, 0),
  p3 = c(0, 2, 0),
  p4 = c(0.5, 1.5, 2),
  p5 = c(0.5, 0.5, 2),
  p6 = c(1.5, 0.5, 2),
  p7 = c(1.5, 1.5, 2)
)
plotHexahedron(hexahedron)
```

rphong_on_hemisphere *Sampling on hemisphere*

Description

Sampling on a hemisphere according to the Phong density (dimension 3).

Usage

```
rphong_on_hemisphere(n, alpha = 0, r = 1)
```

Arguments

n	number of simulations
alpha	parameter of the Phong density, a positive number; 0 for uniform sampling (default)
r	radius

Value

The simulations in a n times 3 matrix.

Examples

```
## Not run:
library(rgl)
sims <- rphong_on_hemisphere(400, alpha = 10)
spheres3d(0, 0, 0, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

---

runif_cube *Uniform sampling on/in cube*

Description

Uniform sampling on or in a cube (arbitrary dimension).

Usage

```
runif_in_cube(n, d, O = rep(0, d), r = 1)

runif_on_cube(n, d, O = rep(0, d), r = 1)
```

Arguments

n	number of simulations
d	dimension
O	center of the cube
r	radius (half-side) of the cube

Value

The simulations in a n times d matrix.

Examples

```
sims <- runif_on_cube(60, d = 2)
plot(sims, xlim = c(-1,1), ylim = c(-1,1), pch = 19, asp = 1)
sims <- runif_in_cube(50, d = 3)
library(scatterplot3d)
scatterplot3d(sims, pch = 19, highlight.3d = TRUE, asp = 1)
```

`runif_ellipsoid` *Uniform sampling on/in ellipsoid*

Description

Uniform sampling on an ellipsoid or in an ellipsoid. The sampling *in* an ellipsoid is available in arbitrary dimension. The sampling *on* an ellipsoid is available only in dimension 2 or 3.

Usage

```
runif_on_ellipse(n, A, r)

runif_on_ellipsoid(n, A, r)

runif_in_ellipsoid(n, A, r)
```

Arguments

`n`	number of simulations
`A`	symmetric positive-definite matrix defining the ellipsoid (see Details), of size 2 for `runif_on_ellipse` and size 2 or 3 for `runif_on_ellipsoid` (for size 2 these are the same functions)
`r`	"radius" (see Details)

Details

The ellipsoid is the set of vectors `x` satisfying `t(x) %*% A %*% x == r^2`. For example, for an axis-aligned ellipse with horizontal radius `a` and vertical radius `b`, take `A=1/diag(c(a^2,b^2))` and `r=1`.

Value

The simulations in a matrix with `n` rows.

Examples

```
library(uniformly)
set.seed(666L)
# ellipse parameters
A <- rbind(c(2, 1), c(1, 1))
r <- 2
# plot the ellipse
x1 <- seq(-2.5, 2.5, length.out = 100)
x2 <- seq(-3, 3, length.out = 100)
z <- outer(
  x1, x2, FUN = Vectorize(function(x1, x2) t(c(x1, x2)) %*% A %*% c(x1, x2))
)
contour(x1, x2, z, nlevels = 1, levels = r^2, asp = 1, drawlabels = FALSE)
# simulations on the perimeter
sims <- runif_on_ellipse(60, A, r)
points(sims, pch = 19, col = "blue")
# simulations in the area
sims <- runif_in_ellipsoid(100, A, r)
points(sims, pch = 19, col = "green")
# 3D example ####
A <- matrix(c(5,1,1, 1,3,1, 1,1,1), ncol = 3L)
r <- 2
# draw the ellipsoid
library(misc3d)
x <- seq(-1, 1, length.out = 50)
y <- seq(-1.5, 1.5, length.out = 50)
z <- seq(-2.7, 2.7, length.out = 50)
g <- as.matrix(expand.grid(x = x, y = y, z = z))
voxel <-
  array(apply(g, 1L, function(v) t(v) %*% A %*% v), dim = c(50, 50, 50))
isosurface <- computeContour3d(voxel, max(voxel), r^2, x = x, y = y, z = z)
drawScene.rgl(makeTriangles(isosurface, alpha = 0.3))
# simulate and plot points on ellipsoid
library(rgl)
sims <- runif_on_ellipsoid(300, A, r)
points3d(sims)
```

`runif_in_annulus` *Uniform sampling in an annulus*

Description

Uniform sampling in an annulus (dimension 2).

Usage

```
runif_in_annulus(n, O, r1, r2)
```

Arguments

n	number of simulations
O	center of the annulus
r1	inner radius
r2	outer radius

Value

The simulations in a n times 2 matrix.

Examples

```
sims <- runif_in_annulus(100, c(0, 0), 1, 2)
plot(sims, xlim = c(-2, 2), ylim = c(-2, 2), asp = 1, pch = 19)
```

---

runif_in_hexahedron *Uniform sampling in a hexahedron*

Description

Uniform sampling in a hexahedron (polyhedron with six faces).

Usage

```
runif_in_hexahedron(n, hexahedron)
```

Arguments

n	number of simulations
hexahedron	a hexahedron given by a 3 times 8 matrix whose eight columns are the vertices; see makeHexahedron

Value

The simulations in a n times 3 matrix.

Examples

```
library(uniformly)
hexahedron <- makeHexahedron(
  p0 = c(0, 0, 0),
  p1 = c(2, 0, 0),
  p2 = c(2, 2, 0),
  p3 = c(0, 2, 0),
  p4 = c(0.5, 1.5, 2),
  p5 = c(0.5, 0.5, 2),
  p6 = c(1.5, 0.5, 2),
```

```
  p7 = c(1.5, 1.5, 2)
)
sims <- runif_in_hexahedron(200, hexahedron)
plotHexahedron(hexahedron, alpha = 0.3)
rgl::points3d(sims)
```

`runif_in_pball` *Uniform sampling in a p-ball*

Description

Uniform sampling in a p-ball (arbitrary dimension).

Usage

```
runif_in_pball(n, d, p, r = 1)
```

Arguments

`n`	number of simulations
`d`	dimension
`p`	exponent in the p-norm, a positive number
`r`	positive number, the radius

Value

The simulations in a `n` times `d` matrix.

Examples

```
sims <- runif_in_pball(500, d = 2, p = 1)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), asp = 1)
```

`runif_in_polygon` *Uniform sampling in a polygon*

Description

Uniform sampling in a polygon (dimension 2).

Usage

```
runif_in_polygon(n, vertices, center = "centroid")
```

Arguments

n	number of simulations
vertices	two-columns matrix giving the vertices (rows); the vertices must be ordered (clockwise or counterclockwise)
center	a point with respect to which the polygon is star-shaped, or "centroid" (default) to take the centroid (see Details)

Details

This function works for a star-shaped polygon, that is, a polygon that contains a point from which the entire polygon boundary is visible. This point must be given in the center argument. If the polygon is convex, any point inside the polygon is suitable (thus the default option of the center argument is appropriate in this case).

Value

The simulations in a n times 2 matrix.

Examples

```
vs <- matrix(c(0.951056516295154, 0.309016994374947,
               0.224513988289793, 0.309016994374947,
               -0.951056516295154, 0.309016994374948,
               -0.363271264002681, -0.118033988749895,
               0.587785252292473, -0.809016994374948,
               0.36327126400268, -0.118033988749895,
               0, 1,
               -0.224513988289793, 0.309016994374947,
               -0.587785252292473, -0.809016994374947,
               0, -0.381966011250105),
             ncol=2, byrow=TRUE)
sims <- runif_in_polygon(500, vs)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), pch = 19, asp = 1)
```

---

runif_in_simplex *Uniform sampling in a simplex*

Description

Uniform sampling in a simplex (arbitrary dimension).

Usage

```
runif_in_simplex(n, simplex)
```

Arguments

n	number of simulations
simplex	a (d+1) times d matrix giving the vertices of the simplex (rows)

Value

The simulations in a n times d matrix.

Note

In dimension 3, you can use runif_in_tetrahedron instead.

Examples

```
simplex <- rbind(c(0,0,0), c(1,0,0), c(1,1,0), c(1,1,2))
sims <- runif_in_simplex(1000, simplex)
library(rgl)
points3d(sims)
```

---

runif_in_tetrahedron *Uniform sampling in a tetrahedron*

Description

Uniform sampling in a tetrahedron (in dimension 3).

Usage

```
runif_in_tetrahedron(n, v1, v2, v3, v4)
```

Arguments

n	number of simulations
v1, v2, v3, v4	vertices of the tetrahedron

Value

The simulations in a n times 3 matrix.

See Also

runif_in_simplex for sampling in a simplex in arbitrary dimension.

Examples

```
library(rgl)
tetrahedron <- tetrahedron3d()
shade3d(tetrahedron, color = "red", alpha = 0.3)
vs <- tetrahedron$vb[1L:3L, ]
sims <- runif_in_tetrahedron(100, vs[, 1], vs[, 2], vs[, 3], vs[, 4])
points3d(sims)
```

`runif_on_spherePatch` *Uniform sampling on a spherical patch*

Description

Uniform sampling on a spherical patch (in dimension 3).

Usage

```
runif_on_spherePatch(n, r = 1, phi1, phi2, theta1, theta2)
```

Arguments

`n`	number of simulations
`r`	radius
`phi1`, `phi2`	numbers defining the latitudinal angle range
`theta1`, `theta2`	numbers defining the longitudinal angle range

Details

A sphere patch is the part of the sphere whose polar angles `theta` and `phi` satisfy `0 <= theta1 <= theta <= theta2 <= 2*pi` and `0 <= phi1 <= phi <= phi2 <= pi`.

Value

The simulations in a `n` times `3` matrix.

See Also

`runif_on_stri` for sampling on a spherical triangle.

Examples

```
# sampling on the first orthant:
sims <-
  runif_on_spherePatch(100, phi1 = 0, phi2 = pi/2, theta1 = 0, theta2 = pi/2)
## Not run:
library(rgl)
spheres3d(0, 0, 0, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

runif_on_sphericalCap *Uniform sampling on a spherical cap*

Description

Uniform sampling on a spherical cap (in dimension 3).

Usage

```
runif_on_sphericalCap(n, r = 1, h)
```

Arguments

n	number of simulations
r	radius of the sphere
h	height of the cap

Value

The simulations in a n times 3 matrix.

Examples

```
sims <- runif_on_sphericalCap(500, r = 2, h = 1)
## Not run:
library(rgl)
spheres3d(0, 0, 0, radius = 2, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

runif_on_stri *Uniform sampling on a spherical triangle*

Description

Uniform sampling on a spherical triangle (in dimension 3).

Usage

```
runif_on_stri(n, r = 1, v1, v2, v3)
```

Arguments

n	number of simulations
r	radius
v1, v2, v3	vertices

Value

The simulations in a n times 3 matrix.

Examples

```
# sampling on the first orthant:
sims <- runif_on_stri(100, v1 = c(1, 0, 0), v2 = c(0, 1, 0), v3 = c(0, 0, 1))
## Not run:
library(rgl)
spheres3d(0, 0, 0, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

---

runif_sphere *Uniform sampling on/in sphere*

Description

Uniform sampling on a sphere or in a sphere, in arbitrary dimension.

Usage

```
runif_on_sphere(n, d, r = 1)

runif_in_sphere(n, d, r = 1)
```

Arguments

n	number of simulations
d	dimension of the space
r	radius of the sphere

Value

The simulations in a n times d matrix.

Examples

```
sims <- runif_on_sphere(20, d = 2)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), asp = 1, pch = 19)
sims <- runif_in_sphere(100, d = 2)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), asp = 1, pch = 19)
```

`runif_torus` *Uniform sampling on/in torus*

Description

Uniform sampling on or in a torus (dimension 3).

Usage

```
runif_on_torus(n, R, r)

runif_in_torus(n, R, r)
```

Arguments

`n`	number of simulations
`R`	major radius
`r`	minor radius

Value

The simulations in a `n` times `3` matrix.

Examples

```
R <- 3; r <- 2
sims_on <- runif_on_torus(50, R = R, r = r)
sims_in <- runif_in_torus(50, R = R, r = r)
library(misc3d)
fx <- function(u,v) (R+r*cos(u)) * cos(v)
fy <- function(u,v) (R+r*cos(u)) * sin(v)
fz <- function(u,v) r*sin(u)
parametric3d(
  fx, fy, fz, umin = 0, umax = 2*pi, vmin = 0, vmax = 2*pi, alpha = 0.3
)
library(rgl)
points3d(sims_on)
points3d(sims_in, color = "red")
```

runif_triangle *Uniform sampling on/in a triangle*

Description

Uniform sampling on or in a triangle (dimension 2).

Usage

```
runif_in_triangle(n, v1, v2, v3)

runif_on_triangle(n, v1, v2, v3)
```

Arguments

n	number of simulations
v1, v2, v3	vertices of the triangle

Value

The simulations in a n times 2 matrix.

Examples

```
sims <- runif_on_triangle(30, c(0,0), c(1,0), c(0,1))
plot(sims, xlim = c(0,1), ylim = c(0,1), pch = 19)
sims <- runif_in_triangle(100, c(0,0), c(1,0), c(0,1))
plot(sims, xlim = c(0,1), ylim = c(0,1), pch = 19)
```

runif_unitSimplex *Uniform sampling on/in a unit simplex*

Description

Uniform sampling on or in a unit simplex (arbitrary dimension).

Usage

```
runif_on_unitSimplex(n, d)

runif_in_unitSimplex(n, d)
```

Arguments

n	number of simulations
d	dimension of the space

Value

The simulations in a n times d matrix.

See Also

runif_in_tetrahedron for sampling in an arbitrary tetrahedron in dimension 3; runif_in_simplex for sampling in an arbitrary simplex.

Examples

```
library(rgl)
sims <- runif_on_unitSimplex(300, d = 3)
points3d(sims)
```

---

surface_sphere *Sphere surface*

Description

Surface of a sphere (arbitrary dimension).

Usage

```
surface_sphere(d, r = 1)
```

Arguments

d	dimension of the space
r	radius of the sphere

Value

The surface of the sphere of radius r in the d-dimensional space.

Examples

```
r <- 2
surface_sphere(3, r)
4*pi*r^2
# perimeter of the unit circle:
surface_sphere(2)
```

surface_spherePatch *Sphere patch surface*

Description

Surface of a sphere patch.

Usage

```
surface_spherePatch(r, phi1, phi2, theta1, theta2)
```

Arguments

r	radius
phi1, phi2	numbers defining the latitudinal angle range
theta1, theta2	numbers defining the longitudinal angle range

Details

A sphere patch is the part of the sphere whose polar angles theta and phi satisfy 0 <= theta1 <= theta <= theta2 <= 2*pi and 0 <= phi1 <= phi <= phi2 <= pi.

Value

The surface of the sphere patch.

See Also

surface_stri for the surface of a spherical triangle.

Examples

```
# surface of the first orthant:
surface_spherePatch(r=1, phi1=0, phi2=pi/2, theta1=0, theta2=pi/2)
surface_stri(r=1, c(1,0,0), c(0,1,0), c(0,0,1))
```

surface_sphericalCap *Spherical cap surface*

Description

Surface of a spherical cap.

Usage

```
surface_sphericalCap(r, h)
```

Arguments

r	radius of the sphere
h	height of the cap

Value

The surface area of the spherical cap.

surface_stri *Spherical triangle surface*

Description

Surface of a spherical triangle.

Usage

```
surface_stri(r, v1, v2, v3)
```

Arguments

r	radius
v1, v2, v3	vertices

Value

The surface of the spherical triangle of radius r with vertices v1, v2, v3.

Examples

```
# surface of the first orthant:
surface_stri(r=1, c(1,0,0), c(0,1,0), c(0,0,1))
```

surface_torus *Torus surface*

Description

Surface of a torus.

Usage

```
surface_torus(R, r)
```

Arguments

R	major radius
r	minor radius

Value

The surface area of the torus.

surface_triangle *Triangle surface*

Description

Surface of a triangle.

Usage

```
surface_triangle(v1, v2, v3)
```

Arguments

v1, v2, v3	vertices of the triangle

Value

The surface of the triangle with vertices v1, v2, v3.

Examples

```
surface_triangle(c(0,0), c(0,1), c(1,0))
```

volume_ellipsoid *Ellipsoid volume*

Description

Volume of an ellipsoid (arbitrary dimension).

Usage

```
volume_ellipsoid(A, r)
```

Arguments

A	symmetric positive-definite matrix defining the ellipsoid (see Details)
r	"radius" (see Details)

Details

The (boundary of the) ellipsoid is the set of vectors `x` satisfying `t(x) %*% A %*% x == r^2`.

Value

The volume of the ellipsoid.

Examples

```
# dimension 2 (area), with diagonal matrix A
A <- diag(c(2,3))
r <- 2
volume_ellipsoid(A, r)
pi * r^2 / sqrt(A[1,1]*A[2,2])
```

volume_hexahedron *Hexahedron volume*

Description

Volume of a hexahedron.

Usage

```
volume_hexahedron(hexahedron)
```

Arguments

hexahedron	a 3 times 8 matrix whose columns are the eight vertices of the hexahedron; see makeHexahedron

Value

The volume of the hexahedron.

Examples

```
library(uniformly)
# a cube with side 2 ####
hexahedron <- makeHexahedron(
  p0 = c(0, 0, 0),
  p1 = c(2, 0, 0),
  p2 = c(2, 2, 0),
  p3 = c(0, 2, 0),
  p4 = c(0, 2, 2),
  p5 = c(0, 0, 2),
  p6 = c(2, 0, 2),
  p7 = c(2, 2, 2)
)
volume_hexahedron(hexahedron) # should be 8
```

---

volume_pball *p-ball volume*

Description

Euclidean volume of a p-ball (arbitrary dimension).

Usage

```
volume_pball(d, p, r = 1)
```

Arguments

d	dimension
p	exponent in the p-norm, a positive number
r	radius of the ball

Value

The volume of the p-ball with radius r.

Examples

```
volume_pball(d=4, p=2, r=2)
volume_sphere(d=4, r=2)
```

volume_simplex *Simplex volume*

Description

Volume of a simplex (arbitrary dimension).

Usage

```
volume_simplex(simplex)
```

Arguments

simplex	a (d+1) times d matrix giving the vertices of the simplex (rows)

Value

The volume of the simplex.

Examples

```
set.seed(666)
simplex <- matrix(rnorm(4*3), nrow=4, ncol=3)
volume_simplex(simplex)
volume_tetrahedron(simplex[1,], simplex[2,], simplex[3,], simplex[4,])
```

volume_sphere *Sphere volume*

Description

Volume of a sphere (arbitrary dimension).

Usage

```
volume_sphere(d, r = 1)
```

Arguments

d	dimension of the space
r	radius of the sphere

Value

The volume of the sphere with radius r in the d-dimensional space.

Examples

```
r <- 2
volume_sphere(3, r)
4/3*pi*r^3
```

`volume_sphericalCap` *Spherical cap volume*

Description

Volume of a spherical cap.

Usage

```
volume_sphericalCap(r, h)
```

Arguments

`r`	radius of the sphere
`h`	height of the cap

Value

The volume of the spherical cap.

`volume_tetrahedron` *Tetrahedron volume*

Description

Volume of a tetrahedron (dimension 3).

Usage

```
volume_tetrahedron(v1, v2, v3, v4)
```

Arguments

`v1`, `v2`, `v3`, `v4`	vertices of the tetrahedron

Value

The volume of the tetrahedron.

See Also

`volume_simplex` for the volume of a simplex in arbitrary dimension.

Examples

```
v1 <- c(0,0,0); v2 <- c(1,0,0); v3 <- c(0,1,0); v4 <- c(0,0,1)
volume_tetrahedron(v1, v2, v3, v4)
volume_unitSimplex(3)
```

`volume_torus` *Torus volume*

Description

Volume of a torus.

Usage

```
volume_torus(R, r)
```

Arguments

`R`	major radius
`r`	minor radius

Value

The volume of the torus.

`volume_unitSimplex` *Unit simplex volume*

Description

Volume of the unit simplex (arbitrary dimension).

Usage

```
volume_unitSimplex(d)
```

Arguments

`d`	dimension of the space

Value

The volume of the unit simplex in the space of dimension `d`.

See Also

`volume_simplex` for the volume of an arbitrary simplex.

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