# Package 'fastbeta'

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fastbeta-package

R Package fastbeta

#### **Description**

An R package for approximating time-varying infectious disease transmission rates from disease incidence time series and other data.

#### **Details**

```
The "main" function is fastbeta.

To render a list of available help topics, use help(package = "fastbeta").

To report a bug or request a change, use bug.report(package = "fastbeta").
```

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cbind.ts

Combine Time Series Objects

# Description

A replacement for the S3 method registered by package **stats** for generic function **cbind** and class **ts**. It sets column names following the rules employed by the internal default method for cbind. It exists to allow users to work around PR#18583, which shows that the method in package **stats** employs different and rather inconsistent rules. This function must be called directly, as it is not registered as a method for cbind.

# Usage

```
## S3 method for class 'ts'
cbind(..., deparse.level = 1)
```

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#### Arguments

```
... vectors (including matrices), at least one inheriting from class ts.

deparse.level an integer (0, 1, or 2) controlling how column names are determined for untagged arguments that are not matrices, following the internal default method
```

for cbind.

#### Value

A "multiple time series" object, inheriting from class mts.

#### **Examples**

```
n <- 3L
x <- matrix(0, n, n, dimnames = list(NULL, LETTERS[seq_len(n)]))</pre>
y <- seq_len(n)
tsx \leftarrow ts(x)
tsy <- ts(y)
`~` <- identity
for (k in 0L:2L) {
cat(sprintf("k = %d:\n\n", k))
withAutoprint({
try(colnames(cbind
                                y, deparse.level = k)))
                     ( x,
try(colnames(cbind ( tsx, tsy, deparse.level = k)))
try(colnames(cbind.ts( tsx, tsy, deparse.level = k)))
try(colnames(cbind (\sim x, \sim y, deparse.level = k)))
try(colnames(cbind (~tsx, ~tsy, deparse.level = k)))
try(colnames(cbind.ts(~tsx, ~tsy, deparse.level = k)))
})
cat("\n\n")
rm(`~`)
```

deconvolve

Richardson-Lucy Deconvolution

# **Description**

Performs a modified Richardson-Lucy iteration for the purpose of estimating incidence from reported incidence or mortality, conditional on a reporting probability and on a distribution of the time to reporting.

# Usage

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#### Arguments

x a numeric vector of length n giving the number of infections or deaths reported

during n successive time intervals of equal duration.

prob a numeric vector of length d+n, d=length(delay)-1, such that prob[d+i] is

the probability that an infection during interval i is eventually reported. prob of

length 1 is recycled.

delay a numeric vector of positive length such that delay[j] is the probability that

an infection during interval i is reported during interval i+j-1, given that it is eventually reported. Hence delay[j] is the probability of a delay by j-1 intervals and d=length(delay)-1 is the maximum delay. delay need not sum

to 1 but must not sum to 0.

start a numeric vector of length d+n, d=length(delay)-1, giving a starting value for

the iteration. start[d+i] estimates the expected number of infections during interval i that are eventually reported. If missing, then a starting value is generated by padding x on the left and right with d-d0 and d0 elements equal to x.pad, choosing d0=which.max(delay)-1. Note that 0 is invariant under the iteration, hence it can be desirable to set x.pad to a small (relative to max(x))

positive number.

x.pad a non-negative number, used only when start is unset; see above.

tol a tolerance indicating a stopping condition; see the reference. Set to 0 if you

want to perform no fewer than iter.max iterations.

iter.max the maximum number of iterations.

complete a logical flag indicating if the result should preserve successive updates to start.

#### **Details**

Do note that temporal alignment of x (length n) and y=deconvolve(x, ...)value (length or number of rows d+n) requires padding x on the left, as in cbind(x=c(rep(NA, d), x), y); see the examples.

#### Value

A list with elements:

value the result of updating start iter times then dividing by prob. If complete

= TRUE, then value is a (d+n)-by-(1+iter) matrix containing start and the

iter successive updates, each divided by prob.

chisq the chi-squared statistics corresponding to value.

iter the number of iterations performed.

subset(value, start == 0) is zero because zero is invariant under the iteration. If delay has 1 leading zeros and t trailing zeros, then head(value, t) and tail(value, 1) are NaN due to divide-by-zero in the iteration. (Conceptually, x and delay provide no information about incidence during those intervals.)

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#### References

Goldstein, E., Dushoff, J., Ma, J., Plotkin, J. B., Earn, D. J. D., & Lipsitch, M. (2009). Reconstructing influenza incidence by deconvolution of daily mortality time series. *Proceedings of the National Academy of Sciences U. S. A.*, 106(51), 21825-21829. doi:10.1073/pnas.0902958106

```
## Example 1: simulation
##
set.seed(2L)
n <- 200L
d <- 50L
p < -0.1
prob <- plogis(rlogis(d + n, location = glogis(p), scale = 0.1))</pre>
delay <- c(0, diff(pgamma(0L:d, 12, 0.4)))
h \leftarrow function (x, a = 1, b = 1, c = 0) a * exp(-b * (x - c)^2)
ans < floor(h(seq(-60, 60, length.out = d + n), a = 1000, b = 0.001))
x0 <- rbinom(d + n, ans, prob)</pre>
x \leftarrow tabulate(rep(1L:(d + n), x0) +
              sample(0L:d, size = sum(x0), replace = TRUE, prob = delay),
              d + n)[-(1L:d)]
str(D0 <- deconvolve(x, prob, delay, complete = FALSE))</pre>
str(D1 <- deconvolve(x, prob, delay, complete = TRUE))</pre>
matplot(-(d - 1L):n,
        cbind(x0, c(rep(NA, d), x), prob * D0[["value"]], p * ans),
        type = c("p", "p", "p", "l"),
        col = c(1L, 1L, 2L, 4L), pch = c(16L, 1L, 16L, NA),
        lty = c(0L, 0L, 0L, 1L), lwd = c(NA, NA, NA, 3),
        xlab = "Time", ylab = "Count")
legend("topleft", NULL,
       c("actual", "actual+delay", "actual+delay+deconvolution", "p*h"),
       col = c(1L, 1L, 2L, 4L), pch = c(16L, 1L, 16L, NA),
       lty = c(0L, 0L, 0L, 1L), lwd = c(NA, NA, NA, 3),
       bty = "n")
plot(0L:D1[["iter"]], D1[["chisq"]],
     xlab = "Iterations", ylab = quote(chi^2))
abline(h = 1, lty = 2L)
##
## Example 2: application to pneumonia and influenza
data(pneumonia, package = "fastbeta")
x <- pneumonia[["series"]][["deaths"]]</pre>
```

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```
delay <- pneumonia[["delay"]][["gpg"]]</pre>
n \leftarrow length(x)
d <- length(delay) - 1L</pre>
r <- 30L
D2 <- deconvolve(x = x, delay = delay, tol = 0, iter.max = r,
                 complete = TRUE)
stopifnot(D2[["iter"]] == r,
          identical(dim(D2[["value"]]), c(d + n, 1L + r)),
          length(D2[["chisq"]]) == 1L + r,
          min(D2[["chisq"]]) < 1)
## Subscript for the first, critical, and last values:
j2 <- c(1L, which.max(D2[["chisq"]] < 1), 1L + r)
matplot(x = seq(from = pneumonia[["series"]][1L, "date"] - d,
                by = 1, length.out = d + n),
        y = cbind(c(rep(NA, d), x), D2[["value"]][, j2]),
        type = "o",
        col = c(1L, 4L, 2L, 3L), pch = 1L, lty = 1L, lwd = 1,
        xlab = "1918", ylab = "deaths")
legend("topleft", NULL,
       c("observed", sprintf("after %d iterations", j2 - 1L)),
       col = c(1L, 4L, 2L, 3L), pch = 1L, 1ty = 1L, 1wd = 1,
       bty = "n")
```

fastbeta

Estimate a Time-Varying Infectious Disease Transmission Rate

## **Description**

Generates a discrete approximation of a time-varying infectious disease transmission rate from an equally spaced disease incidence time series and other data.

#### Usage

#### Arguments

series

a "multiple time series" object, inheriting from class mts, with three columns storing ("parallel", equally spaced) time series of incidence, births, and the per capita natural mortality rate, in that order.

sigma, gamma, delta

non-negative numbers. m\*sigma, n\*gamma, and delta are the rates of removal from each latent, infectious, and recovered compartment.

a non-negative integer indicating a number of latent stages.

m

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n a positive integer indicating a number of infectious stages. init a numeric vector of length 1+m+n+1 giving an initial state with compartments ordered as (S, E, I, R).

optional arguments passed to deconvolve, if the first column of series represents *observed* incidence rather than actual or estimated incidence.

#### **Details**

The algorithm implemented by fastbeta is based on an SEIR model with

- m latent stages  $(E^i, i = 1, \ldots, m)$ ;
- n infectious stages  $(I^j, j = 1, \ldots, n)$ ;
- time-varying rates  $\beta$ ,  $\nu$ , and  $\mu$  of transmission, birth, and natural death; and
- constant rates  $m\sigma$ ,  $n\gamma$ , and  $\delta$  of removal from each latent, infectious, and recovered compartment, where removal from the recovered compartment implies return to the susceptible compartment (loss of immunity).

It is derived by linearizing of the system of ordinary differential equations

$$\begin{array}{lll} {\rm d}S & /{\rm d}t = & \delta R & -(\lambda(t) + \mu(t))S & +\nu(t) \\ {\rm d}E^1 & /{\rm d}t = \lambda(t)S & -(m\sigma + \mu(t))E^1 \\ {\rm d}E^{i+1}/{\rm d}t = & m\sigma E^i & -(m\sigma + \mu(t))E^{i+1} \\ {\rm d}I^1 & /{\rm d}t = & m\sigma E^m - (m\gamma + \mu(t))I^1 \\ {\rm d}I^{j+1}/{\rm d}t = & n\gamma I^j & -(m\gamma + \mu(t))I^{j+1} \\ {\rm d}R & /{\rm d}t = & n\gamma I^n & -(\delta + \mu(t))R \end{array} \right. \\ \end{array}$$

and substituting actual or estimated incidence and births for definite integrals of  $\lambda S$  and  $\nu$ . This procedure yields a system of linear difference equations from which one recovers a discrete approximation of  $\beta$ :

imation of 
$$\beta$$
: 
$$E_{t+1}^1 = [(1 - \frac{1}{2}(m\sigma + \mu_t))E_t^1 + Z_{t+1}]/[1 + \frac{1}{2}(m\sigma + \mu_{t+1})]$$

$$E_{t+1}^{i+1} = [(1 - \frac{1}{2}(m\sigma + \mu_t))E_t^{i+1} + \frac{1}{2}m\sigma(E_t^i + E_{t+1}^i)]/[1 + \frac{1}{2}(m\sigma + \mu_{t+1})]$$

$$I_{t+1}^1 = [(1 - \frac{1}{2}(n\gamma + \mu_t))I_t^1 + \frac{1}{2}m\sigma(E_t^m + E_{t+1}^m)]/[1 + \frac{1}{2}(n\gamma + \mu_{t+1})]$$

$$I_{t+1}^{j+1} = [(1 - \frac{1}{2}(n\gamma + \mu_t))I_t^{j+1} + \frac{1}{2}n\gamma(I_t^j + I_{t+1}^j)]/[1 + \frac{1}{2}(n\gamma + \mu_{t+1})]$$

$$R_{t+1} = [(1 - \frac{1}{2}(\delta + \mu_t))R_t + \frac{1}{2}n\gamma(I_t^n + I_{t+1}^n)]/[1 + \frac{1}{2}(\delta + \mu_{t+1})]$$

$$S_{t+1} = [(1 - \frac{1}{2}(\mu_t))S_t + \frac{1}{2}\delta(R_t + R_{t+1}) - Z_{t+1} + B_{t+1}]/[1 + \frac{1}{2}(\mu_t)]$$

where we use the notation

$$\begin{split} Z(t) &= \int_{t-1}^t \lambda(s) S(s) \, \mathrm{d}s \\ X_t \sim X(t) : X = S, E^i, I^j, R, Z, B, \mu, \beta \\ B(t) &= \int_{t-1}^t \nu(s) \, \mathrm{d}s \end{split}$$

and it is understood that the independent variable t is a unitless measure of time relative to the spacing of the substituted time series of incidence and births.

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#### Value

A "multiple time series" object, inheriting from class mts, with 1+m+n+1+1 columns (named S, E, I, R, and beta) storing the result of the iteration described in 'Details'. It is completely parallel to argument series, having the same tsp attribute.

#### References

Jagan, M., deJonge, M. S., Krylova, O., & Earn, D. J. D. (2020). Fast estimation of time-varying infectious disease transmission rates. *PLOS Computational Biology*, *16*(9), Article e1008124, 1-39. doi:10.1371/journal.pcbi.1008124

# Examples

```
if (requireNamespace("adaptivetau")) withAutoprint({
data(seir.ts02, package = "fastbeta")
a <- attributes(seir.ts02)</pre>
str(seir.ts02)
plot(seir.ts02)
## We suppose that we have perfect knowledge of incidence,
## births, and the data-generating parameters
series <- cbind.ts(seir.ts02[, c("Z", "B")], mu = a[["mu"]](0))
args <- c(list(series = series),</pre>
          a[c("sigma", "gamma", "delta", "m", "n", "init")])
str(args)
X <- do.call(fastbeta, args)</pre>
str(X)
plot(X)
plot(X[, "beta"], ylab = "Transmission rate")
lines(a[["beta"]](time(X)), col = "red") # the "truth"
})
```

fastbeta.bootstrap

Parametric Bootstrapping

# Description

A simple wrapper around fastbeta using it to generate a "primary" estimate of a time-varying transmission rate and r bootstrap estimates. Bootstrap estimates are computed for incidence time series simulated using seir, with transmission rate defined as the linear interpolant of the primary estimate.

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#### Usage

```
fastbeta.bootstrap(r,
                   series, sigma = 1, gamma = 1, delta = 0,
                   m = 1L, n = 1L, init, ...
```

# **Arguments**

r a non-negative integer indicating a number of replications. a "multiple time series" object, inheriting from class mts, with three columns series storing ("parallel", equally spaced) time series of incidence, births, and the per capita natural mortality rate, in that order. sigma, gamma, delta non-negative numbers. m\*sigma, n\*gamma, and delta are the rates of removal from each latent, infectious, and recovered compartment. m a non-negative integer indicating a number of latent stages. a positive integer indicating a number of infectious stages. init a numeric vector of length 1+m+n+1 giving an initial state with compartments ordered as (S, E, I, R).

optional arguments passed to seir and/or deconvolve. Both take optional arguments prob and delay. When prob is supplied but not delay, seir and deconvolve receive prob as is. When both are supplied, seir receives prob as is, whereas deconvolve receives prob augmented with length(delay)-1 ones.

#### Value

A "multiple time series" object, inheriting from class mts, with 1+r columns storing the one primary and r bootstrap estimates. It is completely parallel to argument series, having the same tsp attribute.

```
if (requireNamespace("adaptivetau")) withAutoprint({
data(seir.ts02, package = "fastbeta")
a <- attributes(seir.ts02)</pre>
str(seir.ts02)
plot(seir.ts02)
## We suppose that we have perfect knowledge of incidence,
## births, and the data-generating parameters
series <- cbind.ts(seir.ts02[, c("Z", "B")], mu = a[["mu"]](0))</pre>
args <- c(list(r = 100L, series = series),</pre>
          a[c("sigma", "gamma", "delta", "m", "n", "init")])
str(args)
R <- do.call(fastbeta.bootstrap, args)</pre>
```

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```
str(R)
plot(R)
plot(R, level = 0.95)
})
```

fastbeta.matrix

Calculate Coefficient Matrix for Iteration Step

# **Description**

Calculates the coefficient matrix corresponding to one step of the iteration carried out by fastbeta:

```
y <- c(1, E, I, R, S)
for (pos in seq_len(nrow(series) - 1L)) {
    L <- fastbeta.matrix(pos, series, ...)
    y <- L %*% y
}</pre>
```

#### Usage

#### Arguments

pos an integer indexing a row (but not the last row) of series.

series a "multiple time series" object, inheriting from class mts, with three columns storing ("parallel", equally spaced) time series of incidence, births, and the per capita natural mortality rate, in that order.

sigma, gamma, delta

non-negative numbers. m\*sigma, n\*gamma, and delta are the rates of removal from each latent, infectious, and recovered compartment.

m a non-negative integer indicating a number of latent stages.

n a positive integer indicating a number of infectious stages.

#### Value

A lower triangular matrix of size 1+m+n+1+1.

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#### **Examples**

```
if (requireNamespace("adaptivetau")) withAutoprint({
data(seir.ts02, package = "fastbeta")
a <- attributes(seir.ts02); p <- length(a[["init"]])</pre>
str(seir.ts02)
plot(seir.ts02)
## We suppose that we have perfect knowledge of incidence,
## births, and the data-generating parameters
series <- cbind.ts(seir.ts02[, c("Z", "B")], mu = a[["mu"]](0))
args <- c(list(series = series),</pre>
           a[c("sigma", "gamma", "delta", "init", "m", "n")])
str(args)
X <- unclass(do.call(fastbeta, args))[, seq_len(p)]</pre>
Y \leftarrow Y. \leftarrow cbind(1, X[, c(2L:p, 1L)], departse.level = 2L)
colnames(Y)
args <- c(list(pos = 1L, series = series),</pre>
          a[c("sigma", "gamma", "delta", "m", "n")])
str(args)
L <- do.call(fastbeta.matrix, args)
str(L)
symnum(L != 0)
for (pos in seq_len(nrow(series) - 1L)) {
    args[["pos"]] <- pos</pre>
    L. <- do.call(fastbeta.matrix, args)</pre>
    Y.[pos + 1L, ] <- L. %*% Y.[pos, ]
}
stopifnot(all.equal(Y, Y.))
})
```

pneumonia

Pneumonia and Influenza Mortality in Philadelphia, PA, 1918

# **Description**

Time series of deaths due to pneumonia and influenza in Philadelphia, PA from September 1, 1918 to December 31, 1918, as recorded in the "Special Tables of Mortality" of the U.S. Census Bureau.

# Usage

```
data(pneumonia, package = "fastbeta")
```

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#### **Format**

A named list with 2 components, series and delay. series is a data frame with 122 rows and 2 variables:

date date of the record.

deaths count of deaths due to influenza and pneumonia.

delay is a data frame with 64 rows and 3 variables:

**nday** number of days from infection to death.

goldstein, gpg probabilities, not summing to 1 due to rounding and truncation; see 'Source'.

#### Source

A script generating the pneumonia object is available as system.file("scripts", "pneumonia.R", package = "fastbeta").

series is obtained from Table 2 in the first reference.

delay is obtained from the remaining references. Component goldstein is obtained from Figure 1 in the Supporting Information of Goldstein et al. (2009). Component gpg is obtained from the convolution of two gamma distributions, one for the time from infection to symptom onset fitted to Figure 1 in Moser et al. (1979) and another for the time from symptom onset to death fitted to Chart 2 in Keeton & Cushman (1918).

#### References

U.S. Census Bureau (1920). Special Tables of Mortality from Influenza and Pneumonia: Indiana, Kansas, and Philadelphia, PA. U.S. Department of Commerce. https://www.census.gov/library/publications/1920/demo/1918-mortality-special-tables.html

Goldstein, E., Dushoff, J., Ma, J., Plotkin, J. B., Earn, D. J. D., & Lipsitch, M. (2009). Reconstructing influenza incidence by deconvolution of daily mortality time series. *Proceedings of the National Academy of Sciences U. S. A.*, 106(51), 21825-21829. doi:10.1073/pnas.0902958106

Moser, M. R., Bender, T. R., Margolis, H. S., Noble, G. R., Kendal, A. P., & Ritter, D. G. (1979). An outbreak of influenza aboard a commercial airliner. *Americal Journal of Epidemiology*, *110*(1), 1-6. doi:10.1093/oxfordjournals.aje.a112781

Keeton, R. W. & Cushman, A. B. (1918). The influenza epidemic in Chicago: the disease as a type of toxemic shock. *Journal of the Americal Medical Association*. 71(24), 1962-1967.

ptpi 13

	D. L. D. LT.
ptpi	Peak to Peak Iteration

# Description

Approximates the state of an SEIR model at a reference time from an equally spaced, T-periodic incidence time series and other data. The algorithm relies on a strong assumption: that the incidence time series was generated by the asymptotic dynamics of an SEIR model admitting a locally stable, T-periodic attractor. Hence do interpret with care.

# Usage

```
ptpi(series, sigma = 1, gamma = 1, delta = 0,
    m = 1L, n = 1L, init,
    start = tsp(series)[1L], end = tsp(series)[2L],
    tol = 1e-03, iter.max = 32L,
    backcalc = FALSE, complete = FALSE, ...)
```

# Arguments

rę	guments	
	series	a "multiple time series" object, inheriting from class mts, with three columns storing ("parallel", equally spaced) time series of incidence, births, and the per capita natural mortality rate, in that order.
	sigma, gamma, del	ta
		non-negative numbers. m*sigma, n*gamma, and delta are the rates of removal from each latent, infectious, and recovered compartment.
	m	a non-negative integer indicating a number of latent stages.
	n	a positive integer indicating a number of infectious stages.
	init	a numeric vector of length 1+m+n+1 giving an initial guess for the state at time start.
	start, end	start and end times for the iteration, whose difference should be approximately equal to an integer number of periods. One often chooses the time of the first peak in the incidence time series and the time of the last peak in phase with the first.
	tol	a tolerance indicating a stopping condition; see 'Details'.
	iter.max	the maximum number of iterations.
	backcalc	a logical indicating if the state at time tsp(series)[1] should be back-calculated from the state at time start if that is later.
	complete	a logical indicating if intermediate states should be recorded in an array. Useful mainly for didactic or diagnostic purposes.
		optional arguments passed to deconvolve, if the first column of series repre-

sents observed incidence rather than actual or estimated incidence.

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#### **Details**

ptpi can be understood as an iterative application of fastbeta to a subset of series. The basic algorithm can be expressed in R code as:

```
w <- window(series, start, end); i <- nrow(s); j <- seq_along(init)
diff <- Inf; iter <- 0L
while (diff > tol && iter < iter.max) {
    init. <- init
    init <- fastbeta(w, sigma, gamma, delta, m, n, init)[i, j]
    diff <- sqrt(sum((init - init.)^2) / sum(init.^2))
    iter <- iter + 1L
}
value <- init</pre>
```

Back-calculation involves solving a linear system of equations; the back-calculated result can mislead if the system is ill-conditioned.

#### Value

A list with elements:

value	an approximation of the state at time start or at time tsp(series)[1L], depending on backcalc.
diff	the relative difference between the last two approximations.
iter	the number of iterations performed.
X	if complete = TRUE, then a "multiple time series" object, inheriting from class mts, with dimensions $c(nrow(w), length(value), iter)$ , where $w = window(series, start, end)$ . $x[, , k]$ contains the state at each time(w) in iteration k.

#### References

Jagan, M., deJonge, M. S., Krylova, O., & Earn, D. J. D. (2020). Fast estimation of time-varying infectious disease transmission rates. *PLOS Computational Biology*, *16*(9), Article e1008124, 1-39. doi:10.1371/journal.pcbi.1008124

```
if (requireNamespace("deSolve")) withAutoprint({
    data(seir.ts01, package = "fastbeta")
    a <- attributes(seir.ts01); p <- length(a[["init"]])
    str(seir.ts01)
    plot(seir.ts01)

## We suppose that we have perfect knowledge of incidence,
## births, and the data-generating parameters, except for
## the initial state, which we "guess"
series <- cbind.ts(seir.ts01[, c("Z", "B")], mu = a[["mu"]](0))</pre>
```

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```
plot(series[, "Z"])
start <- 23; end <- 231
abline(v = c(start, end), lty = 2)
set.seed(0L)
args <- c(list(series = series),</pre>
          a[c("sigma", "gamma", "delta", "m", "n", "init")],
          list(start = start, end = end, complete = TRUE))
init <- seir.ts01[which.min(abs(time(seir.ts01) - start)), seq_len(p)]</pre>
args[["init"]] <- init * rlnorm(p, 0, 0.1)</pre>
str(args)
L <- do.call(ptpi, args)
str(L)
S <- L[["x"]][, "S", ]
plot(S, plot.type = "single")
lines(seir.ts01[, "S"], col = "red", lwd = 4) \# the "truth"
abline(h = L[["value"]]["S"], v = start, col = "blue", lwd = 4, lty = 2)
## Relative error
L[["value"]] / init - 1
})
```

seir

Simulate Infectious Disease Time Series

#### **Description**

Simulates incidence time series based on an SEIR model with user-defined forcing and a simple model for observation error.

Note that simulation code depends on availability of suggested packages **adaptivetau** and **deSolve**. If the dependency cannot be loaded then an error is signaled.

#### Usage

```
seir(length.out = 1L,
    beta, nu = function (t) 0, mu = function (t) 0,
    sigma = 1, gamma = 1, delta = 0,
    m = 1L, n = 1L, init,
    stochastic = TRUE, prob = 1, delay = 1,
    aggregate = FALSE, useCompiled = TRUE, ...)

## A basic wrapper for the m=0L case:

sir(length.out = 1L,
    beta, nu = function (t) 0, mu = function (t) 0,
    gamma = 1, delta = 0,
```

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a non-negative integer indicating the time series length.

```
n = 1L, init,
stochastic = TRUE, prob = 1, delay = 1,
aggregate = FALSE, useCompiled = TRUE, ...)
```

# **Arguments**

length.out

beta, nu, mu functions of one or more arguments returning transmission, birth, and natural death rates at the time point indicated by the first argument. Arguments after the first must be strictly optional. The functions need not be vectorized. sigma, gamma, delta non-negative numbers. m\*sigma, n\*gamma, and delta are the rates of removal from each latent, infectious, and recovered compartment. a non-negative integer indicating a number of latent stages. m a positive integer indicating a number of infectious stages. init a numeric vector of length 1+m+n+1 giving an initial state with compartments ordered as (S, E, I, R). a logical indicating if the simulation should be stochastic; see 'Details'. stochastic prob a numeric vector of length n such that prob[i] is the probability that an infection during interval i is eventually observed. prob of length 1 is recycled. delay a numeric vector of positive length such that delay[i] is the probability that an infection during interval j is observed during interval j+i-1, given that it is eventually observed. delay need not sum to 1 but must not sum to 0. aggregate a logical indicating if latent and infectious compartments should be aggregated.

a logical indicating if derivatives should be computed by compiled C functions rather than by R functions (which may be byte-compiled). Set to FALSE only if TRUE seems to cause problems, and in that case please report the problems with bug.report(package = "fastbeta").

optional arguments passed to 1soda (directly) or ssa.adaptivetau (via its list argument tl.params), depending on stochastic.

#### **Details**

useCompiled

Simulations are based on an SEIR model with

- m latent stages  $(E^i, i = 1, \ldots, m)$ ;
- n infectious stages  $(I^j, j = 1, \dots, n)$ ;
- time-varying rates  $\beta$ ,  $\nu$ , and  $\mu$  of transmission, birth, and natural death; and
- constant rates  $m\sigma$ ,  $n\gamma$ , and  $\delta$  of removal from each latent, infectious, and recovered compartment, where removal from the recovered compartment implies return to the susceptible compartment (loss of immunity).

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seir(stochastic = FALSE) works by numerically integrating the system of ordinary differential equations

$$\begin{array}{lll} {\rm d}S & /{\rm d}t = & \delta R & -(\lambda(t) + \mu(t))S & +\nu(t) \\ {\rm d}E^1 & /{\rm d}t = \lambda(t)S & -(m\sigma + \mu(t))E^1 \\ {\rm d}E^{i+1}/{\rm d}t = & m\sigma E^i & -(m\sigma + \mu(t))E^{i+1} \\ {\rm d}I^1 & /{\rm d}t = & m\sigma E^m - (m\gamma + \mu(t))I^1 \\ {\rm d}I^{j+1}/{\rm d}t = & n\gamma I^j & -(m\gamma + \mu(t))I^{j+1} \\ {\rm d}R & /{\rm d}t = & n\gamma I^n & -(\delta + \mu(t))R \end{array} \right. \\ \end{array}$$

where it is understood that the independent variable t is a unitless measure of time relative to an observation interval. To get time series of incidence and births, the system is augmented with two equations describing *cumulative* incidence and births

$$dZ/dt = \lambda(t)S$$
$$dB/dt = \nu(t)$$

and the *augmented* system is numerically integrated. Observed incidence is simulated from incidence by scaling the latter by prob and convolving the result with delay.

seir(stochastic = TRUE) works by simulating a Markov process corresponding to the augmented system, as described in the reference. Observed incidence is simulated from incidence by binning binomial samples taken with probabilities prob over future observation intervals according to multinomial samples taken with probabilities delay.

#### Value

A "multiple time series" object, inheriting from class mts. Beneath the class, it is a length outby-(1+m+n+1+2) numeric matrix with columns S, E, I, R, Z, and B, where Z and B specify incidence and births as the number of infections and births since the previous time point.

If prob or delay is not missing, then there is an additional column Z.obs specifying *observed* incidence as the number of infections observed since the previous time point. The first length(delay) elements of this column contain partial counts.

# References

Cao, Y., Gillespie, D. T., & Petzold, L. R. (2007). Adaptive explicit-implicit tau-leaping method with automatic tau selection. *Journal of Chemical Physics*, 126(22), Article 224101, 1-9. doi:10.1063/1.2745299

#### See Also

```
seir.auxiliary, seir.library.
```

```
if (requireNamespace("adaptivetau")) withAutoprint({
beta <- function (t, a = 1e-01, b = 1e-05) b * (1 + a * sinpi(t / 26))</pre>
```

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```
<- function (t) 1e+03
nu
    <- function (t) 1e-03
mu
sigma <- 0.5
gamma <- 0.5
delta <- 0
init <- c(S = 50200, E = 1895, I = 1892, R = 946011)
length.out <- 250L</pre>
prob <- 0.1
delay \leftarrow diff(pgamma(0:8, 2.5))
set.seed(0L)
X <- seir(length.out, beta, nu, mu, sigma, gamma, delta, init = init,</pre>
          prob = prob, delay = delay, epsilon = 0.002)
##
## default epsilon = 0.05 allows too big leaps => spurious noise
##
str(X)
plot(X)
r <- 10L
Y <- do.call(cbind.ts, replicate(r, simplify = FALSE,
seir(length.out, beta, nu, mu, sigma, gamma, delta, init = init,
     prob = prob, delay = delay, epsilon = 0.002)[, "Z.obs"]))
str(Y)
plot(window(Y, start = tsp(Y)[1L] + length(delay) / tsp(Y)[3L]),
               ^ ^ ^ ^ ^
     ## discards points showing edge effects due to 'delay'
     plot.type = "single", col = seq_len(r), ylab = "Case reports")
})
```

seir.auxiliary

Auxiliary Functions for the SEIR Model without Forcing

#### **Description**

Calculate the basic reproduction number, endemic equilibrium, and Jacobian matrix of the SEIR model without forcing.

# Usage

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#### **Arguments**

beta, nu, mu, sigma, gamma, delta

non-negative numbers. beta, nu, and mu are the rates of transmission, birth, and natural death. m\*sigma, n\*gamma, and delta are the rates of removal from each latent, infectious, and recovered compartment.

m a non-negative integer indicating a number of latent stages.

n a positive integer indicating a number of infectious stages.

N a non-negative number indicating a population size for the (nu == 0 & mu == 0) case.

#### **Details**

If  $\mu, \nu = 0$ , then the basic reproduction number is computed as

$$\mathcal{R}_0 = N\beta/\gamma$$

and the endemic equilibrium is computed as

$$\begin{bmatrix} S \\ E^i \\ I^j \\ R \end{bmatrix} = \begin{bmatrix} \gamma/\beta \\ w\delta/(m\sigma) \\ w\delta/(n\gamma) \\ w \end{bmatrix}$$

where w is chosen so that the sum is N.

If  $\mu, \nu > 0$ , then the basic reproduction number is computed as

$$\mathcal{R}_0 = \nu \beta a^{-m} (1 - b^{-n}) / \mu^2$$

and the endemic equilibrium is computed as

$$\begin{bmatrix} S \\ E^{i} \\ I^{j} \\ R \end{bmatrix} = \begin{bmatrix} \mu a^{m}/(\beta(1-b^{-n})) \\ wa^{m-i}b^{n}(\delta+\mu)/(m\sigma) \\ wb^{n-j}(\delta+\mu)/(n\gamma) \\ w \end{bmatrix}$$

where w is chosen so that the sum is  $\nu/\mu$ , the population size at equilibrium, and  $a=1+\mu/(m\sigma)$  and  $b=1+\mu/(n\gamma)$ .

Currently, none of the functions documented here are vectorized. Arguments must have length 1.

#### Value

seir.R0 returns a numeric vector of length 1. seir.ee returns a numeric vector of length 1+m+n+1. seir.jacobian returns a function of one argument x (which must be a numeric vector of length 1+m+n+1) whose return value is a square numeric matrix of size length(x).

# See Also

seir, for the system of ordinary differential equations on which these computations are predicated.

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seir.library

Often Used Simulations

#### **Description**

Infectious disease time series simulated using seir, for use primarily in examples, tests, and vignettes. Users should not rely on simulation details, which may change between package versions.

Note that simulation code depends on availability of suggested packages **adaptivetau** and **deSolve**. If the dependency cannot be loaded then the value of the data set is NULL.

# Usage

```
## if (requireNamespace("deSolve"))
data(seir.ts01, package = "fastbeta")
## else ...

## if (requireNamespace("adaptivetau"))
data(seir.ts02, package = "fastbeta")
## else ...
```

#### **Format**

A "multiple time series" object, inheriting from class mts, always a subset of the result of a call to seir, discarding transient behaviour. Simulation parameters may be preserved as attributes.

#### **Source**

Scripts sourced by data to reproduce the simulations are located in subdirectory 'data' of the **fastbeta** installation; see, e.g. system.file("data", "seir.ts01.R", package = "fastbeta").

#### See Also

seir.

```
if (requireNamespace("deSolve")) withAutoprint({
  data(seir.ts01, package = "fastbeta")
  str(seir.ts01)
  plot(seir.ts01)
})

if (requireNamespace("adaptivetau")) withAutoprint({
  data(seir.ts02, package = "fastbeta")
  str(seir.ts02)
```

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```
plot(seir.ts02)
})
```

sir.aoi

Solve the SIR Equations Structured by Age of Infection

# Description

Numerically integrates the SIR equations with rates of transmission and recovery structured by age of infection.

#### Usage

```
sir.aoi(from = 0, to = from + 1, by = 1,
    R0, ell = 1, eps = 0, n = max(length(R0), length(ell)),
    init = c(1 - init.infected, init.infected),
    init.infected = .Machine[["double.neg.eps"]],
    weights = rep(c(1, 0), c(1L, n - 1L)),
    F = function (x) 1, Fargs = list(),
    H = identity, Hargs = list(),
    root = NULL, root.max = 1L, root.break = TRUE,
    aggregate = FALSE, skip.Y = FALSE, ...)

## S3 method for class 'sir.aoi'
summary(object, name = "Y", tol = 1e-6, ...)
```

# **Arguments**

from, to, by	passed to seq.int in order to generate an increasing, equally spaced vector of time points in units of the mean time spent infectious.
RØ	a numeric vector of length n such that $sum(R0)$ is the basic reproduction number and $R0[j]$ is the contribution of infected compartment j. Otherwise, a numeric vector of length 1, handled as equivalent to $rep(R0/n, n)$ .
ell	a numeric vector of length n such that ell[j] is the ratio of the mean time spent in infected compartment j and the mean time spent infectious; internally, ell/sum(ell[R0 > 0]) is used, hence ell is determined only up to a positive factor. Otherwise (and by default), a numeric vector of length 1, handled as equivalent to rep(1, n).
eps	a non-negative number giving the the ratio of the mean time spent infectious and the mean life expectancy; $eps = 0$ implies that life expectancy is infinite (that there are no deaths).
n	a positive integer giving the number of infected compartments. Setting n and thus overriding the default expression is necessary only if the lengths of $R0$ and $ell$ are both 1.
init	a numeric vector of length 2 giving initial susceptible and infected proportions.

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init.infected	a number in $(0,1]$ used only to define the default expression for init; see 'Usage'.
weights	a numeric vector of length n containing non-negative weights, defining the initial distribution of infected individuals among the infected compartments. By default, all infected individuals occupy the first compartment.
F, H	functions returning a numeric vector of length 1 or of length equal that of the first formal argument. The body must be symbolically differentiable with respect to the first formal argument; see D.
Fargs, Hargs	lists of arguments passed to F or H. In typical usage, F and H define parametric families of functions of one variable, and Fargs and Hargs supply parameter values. For example: $H = function(x, h) x^h$ , $Hargs = list(h = 0.996)$ .
root	a function returning a numeric vector of length 1, with formal arguments (tau, S, I, Y, dS, dI, dY, R0, ell) (or a subset); otherwise, NULL. Roots of this function in the interval from from to to are sought alongside the numerical solution.
root.max	a positive integer giving a stopping condition for the root finder. Root finding continues until the count of roots found is root.max.
root.break	a logical indicating if the solver should stop once root.max roots are found. If TRUE, then the numerical solution ends at the last time point less than or equal to the last root.
aggregate	a logical indicating if infected compartments should be aggregated.
skip.Y	a logical indicating if solution of the equation for $Y$ should not be attempted, e.g., because that equation seems stiffer than the rest.
	optional arguments passed to the solver, function 1soda in package <b>deSolve</b> .
object	an R object inheriting from class $\verb"sir.aoi"$ , typically the value of a call to $\verb"sir.aoi"$ .
name	a character string in colnames(object). Tail exponents of $V$ are approximated, where, for example, $V=Y$ if name = "Y" and $V=\sum_j I_j$ (prevalence) if name = "I".
tol	a positive number giving an upper bound on the relative change (from one time point to the next) in the slope of $\log(V)$ , defining time windows in which growth or decay of $V$ is considered to be exponential.

# **Details**

The SIR equations with rates of transmission and recovery structured by age of infection are

$$\begin{split} \mathrm{d}S & /\mathrm{d}t = \mu(1-S) - (\sum_j \beta_j F I_j) H(S) \\ \mathrm{d}I_1 & /\mathrm{d}t = (\sum_j \beta_j F I_j) H(S) - (\gamma_1 + \mu) I_1 \\ \mathrm{d}I_{j+1} /\mathrm{d}t = \gamma_j I_j - (\gamma_{j+1} + \mu) I_{j+1} \\ \mathrm{d}R & /\mathrm{d}t = \gamma_n I_n - \mu R \end{split}$$

where  $S, I_j, R \ge 0, S + \sum_j I_j + R = 1$ , F is a forcing function, and H is a susceptible heterogeneity function. In general, F and H are nonlinear. In the standard SIR equations, F is 1 and H is the identity function.

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Nondimensionalization using parameters  $\mathcal{R}_{0,j} = \beta_j/(\gamma_j + \mu)$ ,  $\ell_j = (1/(\gamma_j + \mu))/t_+$ , and  $\varepsilon = t_+/(1/\mu)$  and independent variable  $\tau = t/t_+$ , where  $t_+ = \sum_{j:\mathcal{R}_{0,j}>0} 1/(\gamma_j + \mu)$  designates as a natural time unit the mean time spent infectious, gives

$$\begin{split} \mathrm{d}S & / \mathrm{d}\tau = \varepsilon (1-S) - (\sum_j (\mathcal{R}_{0,j}/\ell_j) F I_j) H(S) \\ \mathrm{d}I_1 & / \mathrm{d}\tau = (\sum_j (\mathcal{R}_{0,j}/\ell_j) F I_j) H(S) - (1/\ell_1 + \varepsilon) I_1 \\ \mathrm{d}I_{j+1}/\mathrm{d}\tau &= (1/\ell_j) I_j - (1/\ell_{j+1} + \varepsilon) I_{j+1} \\ \mathrm{d}R & / \mathrm{d}\tau = (1/\ell_n) I_n - \varepsilon R \end{split}$$

sir.aoi works with the nondimensional equations, dropping the last equation (which is redundant given  $R=1-S-\sum_j I_j$ ) and augments the resulting system of 1+n equations with a new equation

$$dY/d\tau = \left(\sum_{j} (\mathcal{R}_{0,j}/\ell_j) F I_j\right) (H(S) - 1/(\sum_{j} \mathcal{R}_{0,j} F))$$

due to the usefulness of the solution Y in applications.

#### Value

A "multiple time series" object, inheriting from class sir.aoi and transitively from class mts, storing the numerical solution. Beneath the class, it is a T-by-(1+n+1) numeric matrix of the form cbind(S, I, Y), T <= length(seq(from, to, by)).

If root is a function, then an attribute root.info of the form list(tau, state = cbind(S, I, Y)) stores the first K roots of that function and the state of the system at each root,  $K \le root.max$ .

If aggregate = TRUE, then infected compartments are aggregated so that the number of columns named I is 1 rather than n. This column stores prevalence, the proportion of the population that is infected. For convenience, there are 5 additional columns named I.E, I.I, foi, inc, and crv. These store the non-infectious and infectious components of prevalence (so that I.E + I.I = I), the force of infection, incidence (so that foi \* S = inc), and the curvature of Y.

The method for summary returns a numeric vector of length 2 containing the left and right "tail exponents" of the variable V indicated by name, defined as the asymptotic values of the slope of  $\log(V)$ . NaN elements indicate that a tail exponent cannot be approximated because the time window represented by object does not cover enough of the tail, where the meaning of "enough" is set by tol.

#### Note

sir.aoi is not a special case of sir nor a generalization. The two functions were developed independently and for different purposes: sir.aoi to validate analytical results concerning the SIR equations as formulated here, sir to simulate incidence time series suitable for testing fastbeta.

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```
str(soln.peak)
info.peak <- attr(soln.peak, "root.info")</pre>
to <- 4 * info.peak[["tau"]] # a more principled endpoint
soln <- sir.aoi(to = to, by = by, R0 = R0, ell = ell,</pre>
                aggregate = TRUE, atol = 0, rtol = 1e-12)
                                  ^ ^ ^ ^
## 'atol', 'rtol', ... are passed to deSolve::lsoda
head(soln)
tail(soln)
plot(soln) # dispatching stats:::plot.ts
plot(soln, log = "y")
(lamb <- summary(soln)) # left and right tail exponents</pre>
xoff \leftarrow function(x, k) x - x[k]
k \leftarrow c(16L, nrow(soln)) \# c(1L, nrow(soln)) worse due to transient
plot(soln[, "Y"], log = "y", ylab = "Y")
abline(v = info.peak[["tau"]], h = info.peak[["state"]][, "Y"],
       lty = 2, lwd = 2, col = "red")
for (i in 1:2)
lines(soln[k[i], "Y"] * exp(lamb[i] * xoff(time(soln), k[i])),
      lty = 2, lwd = 2, col = "red")
wrap <-
function (root, ...)
attr(sir.aoi(to = to, by = by, R0 = R0, ell = ell,
             root = root, aggregate = TRUE, ...),
     "root.info")[["tau"]]
Ymax <- info.peak[["state"]][, "Y"]</pre>
## NB: want *simple* roots, not *multiple* roots
L \leftarrow list(function (Y) (Y - Ymax * 0.5) ,
          function (Y) (Y - Ymax * 0.5)^2,
          function (Y) (Y - Ymax
                                   ) ,
          function (Y) (Y - Ymax
                                      )^2)
lapply(L, wrap)
## NB: critical values can be attained more than once
function (Y, dY) if (dY > 0) Y - Ymax * 0.5 else 1,
          function (Y, dY) if (dY < 0) Y - Ymax * 0.5 else 1)
lapply(L, wrap, root.max = 2L)
})
```

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#### **Description**

Time series of deaths due to smallpox, deaths due to all causes, and births in London, England, from 1661 to 1930, as recorded in the London Bills of Mortality and the Registrar General's Weekly Returns.

#### Usage

```
data(smallpox, package = "fastbeta")
```

#### **Format**

A data frame with 13923 observations of 5 variables:

from start date of the record.

**nday** length of the record, which is the number of days (typically 7) over which deaths and births were counted.

smallpox count of deaths due to smallpox.

allcauses count of deaths due to all causes.

births count of births.

#### **Source**

A script generating the smallpox object from a CSV file accompanying the reference is available as system.file("scripts", "smallpox.R", package = "fastbeta").

A precise description of the data set and its correspondence to the original source documents is provided in the reference.

# References

Krylova, O. & Earn, D. J. D. (2020). Patterns of smallpox mortality in London, England, over three centuries. *PLOS Biology*, *18*(12), Article e3000506, 1-27. doi:10.1371/journal.pbio.3000506

```
data(smallpox, package = "fastbeta")
str(smallpox)
table(smallpox[["nday"]]) # not all 7 days, hence:
plot(7 * smallpox / as.double(nday) ~ from, smallpox, type = "1")
```

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