# Package 'emplikAUC'

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Version 0.5
<b>Title</b> Empirical Likelihood Ratio Test/Confidence Interval for AUC or pAUC
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<b>Depends</b> R (>= 3.2.5), emplik2, rootSolve
Imports stats
Description Test hypotheses and construct confidence intervals for AUC (area under Receiver Operating Characteristic curve) and pAUC (partial area under ROC curve), from the given two samples of test data with disease/healthy subjects. The method used is based on TWO SAMPLE empirical likelihood and PROFILE empirical likelihood, as de scribed in <a href="https://www.ms.uky.edu/~mai/research/eAUC1.pdf">https://www.ms.uky.edu/~mai/research/eAUC1.pdf</a> >.
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Description

EL2paucT1T2

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : pAUC(p1, p2) = theta; F(tau1)=1-p2; F(tau2)=1-p1. The two samples are in the x-vector and y-vector.

Testing one pAUC(p1, p2) and two quantile values together by Empir-

### Usage

EL2paucT1T2(tauVec, pauc, partial1, partial2, x, y, epsxy)

ical Likelihood.

### **Arguments**

tauVec	The vector (tau1, tau2), the two quantile values, to be tested.
pauc	The value of the pAUC(p1, p2) under $H_0$ , to be tested.
partial1	The probability that define the quantile 1
partial2	The probability that define the quantile 2. Must satisfy partial $1 < \text{partial } 2$ .
x	a vector of observations, length m, for the first sample.
у	a vector of observations, length n, for the second sample.
epsxy	The parameter for smoothing when compare x-y.

#### **Details**

The input tauVec=(tau1, tau2), and must have: tau1 < tau2. The relavant definitions are:  $tau1 = F^{-1}(1-partial2)$ ;  $tau2 = F^{-1}(1-partial1)$ . Thus, we must have partial2 > partial1.

This function is testing 3 parameters simultanuously. It depend on the package emplik2.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j \; ; \quad s.t. \quad \sum v_i = 1 \; , \quad \sum \nu_j = 1 \; .$$

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### Value

A single value that is the "-2LLR" from emplik2::el2.cen.EMm(). Typically should be distributed as chi square df=3, under  $H_0$ .

### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

el2test4auc

Testing one AUC value by Empirical likelihood.

### **Description**

By calling upon a function from the package emplik2 (using EM), this function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : AUC = theta. The two samples are in the x-vector and y-vector inputs.

### Usage

```
el2test4auc(theta, x, y, ind)
```

### Arguments

theta	The "true" value of the AUC under $H_0$ , to be tested.
x	a vector of observations, length m, for the first sample. The test-results of healthy subjects.
У	a vector of observations, length n, for the second sample. The test-results of desease subjects.
ind	A smoothed indicator function, to generate a Matrix of (smoothed) indicator values: $I[x[i] < y[j]]$ .

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#### **Details**

This function is similar to the function eltest4aucONE(), the difference is that we call the function emplik2::el2.cen.EMs() to do the heavy computation (instead of by our own code). So, the speed and convergence property may be slightly different. When they both converge the results should be the same.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j ; \quad \sum v_i = 1 , \quad \sum \nu_j = 1 .$$

#### Value

A list that is the same as el2.cen.EMs() from emplik2 package. Which contains

lambda The final tilting parameter.

"-2LLR" The -2 log empirical likelihood ratio.

Pval The p-value.

iterNum The iteration number used in computing.

### Author(s)

Mai Zhou <maizhou@gmail.com>.

### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

#### **Examples**

```
 y <- c(10, 209, 273, 279, 324, 391, 566, 785) \\ x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524) \\ \#\#\# \text{ The estimation of AUC} \\ sum(smooth3(x=x, y=y))/(length(x)*length(y)) \\ \#\#\# \text{ This does not work in Rcmd check: (truncate at %*%)} \\ \#\#\# \text{ rep}(1/length(x), length(x))%*%smooth3(x=x, y=y)%*%rep(1/length(y), length(y))} \\ \#\#\# \text{ The result should be 0.75.} \\ \#\#\# \text{ We may then test a hypothesis about the AUC value: H0: AUC= 0.7} \\ el2test4auc(theta=0.7, x=x, y=y, ind=smooth3) \\ \#\#\#\# \text{ Two of the outputs should be '-2LLR'=0.1379561 and Pval=0.7103214}
```

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el2testPauc	Testing one pAUC(0, p) value by Empirical likelihood.	

#### **Description**

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : pAUC(0, p) = theta. The two samples are in the x-vector and y-vector inputs.

### Usage

```
el2testPauc(theta, x, y, ind, nuilow, nuiup, partial, epsxy, epsT)
```

#### **Arguments**

theta	The "true" value of the pAUC $(0, p)$ under $H_0$ , to be tested.
x	a vector of observations, length m, for the first sample, test-results with the healthy subjects.
У	a vector of observations, length n, for the second sample, test-results with the desease subjects.
ind	The (smoothed) indicator function for compare x-y.
nuilow	Lower bound for the nuisamce parameter (1-p)-th quantile of X) search.
nuiup	Upper bound for nuisance parameter search.
partial	The probability $p$ in pAUC(0, $p$ ).
epsxy	The smoothing parameter when compare x-y.
epsT	The smoothing parameter when calculating quantile.

### **Details**

This function will call another function: el2testPaucT(). We then use optimize() to profile out the nuisance parameter tau: the (1-p)-th quantile of X distribution.

Can be used by findUnew() etc.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j \; ; \quad s.t. \quad \sum v_i = 1 \; , \quad \sum \nu_j = 1 \; .$$

### Value

A list containing

"-2LLR" The -2 log empirical likelihood ratio.

Nupar The nuisance parameter value that achieved the minimum.

Pval The p-value, by using chi square distribution with 1 df.

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#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

el2testPaucT	Testing one $pAUC(0, p)$ value and one quantile: $F(tau) = 1$ -p together
	by Empirical Likelihood.

### Description

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : pAUC(0, p) = theta and F(tau) = 1-p. (F is the CDF of X). The two samples data are in the x-vector and y-vector inputs. It uses EM.

### Usage

```
el2testPaucT(tau, pauc, ind, partial, x, y, epsxy, epsT)
```

### **Arguments**

tau	The "true" value of the (1-p)-th quantile of X-distribution F, to be tested.
pauc	The $H_0$ value of pAUC(0, p) to be tested.
ind	A smoothed indicator function, to generate a Matrix of (smoothed) indicator values: $I[x[i] < y[j]]$ .
partial	The probability p in pAUC(0, p); also the p in $F(tau) = 1-p$ .
x	a vector of observations, length m, for the first sample, test-results with healthy subjects.
У	a vector of observations, length n, for the second sample, test-results with desease subjects.
epsxy	The smoothing parameter when compare x-y.
epsT	The smoothing parameter when compare x to Tau, for quantile estimation.

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#### **Details**

This function is called by el2testPauc(). It is listed here stand alone because users may find it useful elsewhere.

It make use of function smooth3() and the function el2.cen.EMm() from the emplik2 package.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j ; \qquad \sum v_i = 1 , \quad \sum \nu_j = 1 .$$

#### Value

It returns one value that is the "-2LLR". It should have chi square df=2 under  $H_0$ .

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

eltest4auc0NE

Testing one AUC value by Empirical likelihood.

### **Description**

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : AUC = theta. The two samples are in the x-vector and y-vector.

#### Usage

```
eltest4aucONE(theta, x, y, ind, tol.u, tol.v, tol.H0)
```

### Arguments

theta	The "true" value of the AUC under $H_0$ , to be tested.
X	a vector of observations, length m, for the first sample. The test-results of healthy subjects
У	a vector of observations, length n, for the second sample. The test-results of desease subjects.

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ind	A smoothed indicator function, to generate a Matrix of (smoothed) indicator values: $I[x[i] < y[j]]$ .
tol.u	Error tol for final u probability vector. Must $> 0$ .
tol.v	Error tol for final v probability vector. Must $> 0$ .
tol.H0	The error bound for the constrained NPMLE to satisfy $H_0$ , must >0.

#### **Details**

This function is similar to el2test4auc, but using our own algorithm (not EM). It may be slightly different to the above in terms of speed and convergence property. We listed 3 kind of tol to control convergence.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j ; \qquad \sum v_i = 1 , \quad \sum \nu_j = 1 .$$

#### Value

#### A list containing

1 amb da	The	fma1	4:14im ~	monomo otom
lambda	THE	IIIIai	unung	parameter.

u the new u vector.
v The new v vector.

"-2LLR" The -2 log empirical likelihood ratio.

Pval The p-value.

iterNum The iteration number used in computing.

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <-c(10, 209, 273, 279, 324, 391, 566, 785)

x <-c(21, 38, 39, 51, 77, 185, 240, 289, 524)

#### We know the AUC estimator here is 0.75.

#### We may test a hypothesis about the AUC value: H0: AUC= 0.7

eltest4aucONE(theta=0.7, x=x, y=y, ind=smooth3, tol.u=1e-6, tol.v=1e-6, tol.H0=1e-6)

#### Two of the outputs should be '-2LLR'=0.1379561 and Pval=0.7103214
```

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eltest4diff2auc	Testing the Difference of 2 AUC values, Paired Data, by Empirical likelihood.

### **Description**

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : AUC1-AUC2 = theta. The two samples of paired data are in the x-matrix(mx2) and y-matrix(nx2).

### Usage

```
eltest4diff2auc(theta, x, y, ind, tol.u, tol.v, tol.H0)
```

### Arguments

theta	The "true" value of the AUC under $H_0$ , to be tested.
x	a matrix of observations, dim $mx2$ , for the first sample. The test-results of biomarker 1 and 2 on healthy subjects
У	a matrix of observations, dim nx2, for the second sample. The test-results of biomarker 1 and 2 on desease subjects.
ind	A (smoothed) indicator function, to generate a Matrix of (smoothed) indicator values: $I[x[i] < y[j]]$ .
tol.u	Error tol for final u probability vector. Must $> 0$ .
tol.v	Error tol for final v probability vector. Must $> 0$ .
tol.H0	The error bound for the constrained NPMLE to satisfy $H_0$ , must >0.

#### **Details**

This function is similar to el2test4auc, but for the difference of 2 AUCs. Using our own algorithm (not EM). We listed 3 kind of tol to control convergence.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j ; \qquad \sum v_i = 1 , \quad \sum \nu_j = 1 .$$

### Value

A list containing

lambda	The final tilting parameter.
u	the new u vector.
V	The new v vector.
"-2LLR"	The -2 log empirical likelihood ratio.
Pval	The p-value.
Maxiter	The iteration number used in computing.

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#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <-c(10, 209, 273, 279, 324, 391, 566, 785) x <-c(21, 38, 39, 51, 77, 185, 240, 289, 524) #### We know the AUC estimator here is 0.75. #### We may test a hypothesis about the AUC value: H0: AUC= 0.7 eltest4aucONE(theta=0.7, x=x, y=y, ind=smooth3, tol.u=1e-6, tol.v=1e-6, tol.H0=1e-6) #### Two of the outputs should be '-2LLR'=0.1379561 and Pval=0.7103214
```

eltest4pauc0NE

Testing one pAUC value by Empirical likelihood.

### **Description**

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : pAUC(0, p) = theta. The two samples are in the x-vector and y-vector.

### Usage

```
eltest4pauc0NE(theta,x,y,nuilow,nuiup,ind,partial,epsxy=0.05,epsT=(length(x))^{-0.75})
```

#### **Arguments**

theta	The "true" value of the pAUC under $H_0$ , to be tested.
X	a vector of observations, length m, for the first sample. Test-results with healthy subjects.
У	a vector of observations, length n, for the second sample. Test-results with desease subjects.
nuilow	The lower bound for the nuisance parameter (the (1-p)-th quantile of X CDF F) search.
nuiup	The upper bound for the nuisance parameter search.
ind	A smoothed indicator function, to generate a Matrix of (smoothed) indicator values: $I[x[i] < y[j]]$ .
partial	The probability $p$ in the pAUC(0, $p$ ).
epsxy	Window width for the smoother, "ind", when compare x-y.
epsT	Window width for the smoother, "ind", when define quantile.

eltest4paucT

#### **Details**

This function calls the function eltest4paucT. We then use optimize() to profile out the nuisance parameter. Return an empirical likelihood ratio siutable for testing one parameter pAUC(0,p).

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j ; \qquad \sum v_i = 1 , \quad \sum \nu_j = 1 .$$

#### Value

A list containing

"-2LLR" The -2 log empirical likelihood ratio.

Nupar The nuisance parameter value to achieve the minimum.

Pval The p-value.

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

eltest4paucT

Testing one pAUC and one quantile together by Empirical Likelihood.

### Description

This function computes the two sample Log Empirical Likelihood ratio for testing  $H_0$ : pAUC(0,p) = theta and F(tau) = 1-p. The two samples are in the x-vector and y-vector.

#### Usage

```
eltest4paucT(tau, x, y, true, ind, epsxy, epsT, tol.u, tol.v, tol.H0, p)
```

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### **Arguments**

tau	The "true" value of the $(1-p)$ th quantile of X, under $H_0$ , to be tested.
x	a vector of observations, length m, for the first sample. Test-results with healthy subjects.
У	a vector of observations, length n, for the second sample. Test-results with desease subjects.
true	The $H_0$ value of the pAUC(0, p) to be tested.
ind	A smoothed indicator function, to generate a Matrix of (smoothed) indicator values: $I[x[i] < y[j]]$ .
epsxy	Window width for smoother (ind) when compare x-y.
epsT	Window width for smoother (ind) when find quantile.
tol.u	Error tol for final u probability vector. Must $> 0$ .
tol.v	Error tol for final v probability vector. Must $> 0$ .
tol.H0	The error bound for checking if the constrained NPMLE satisfy H0, must >0.
p	The probability p in pAUC(0, p), and also in $F(tau) = 1-p$ .

#### **Details**

This function is similar to el2testPaucT(). Just a different algorithm (not EM). Speed and convergence may be slightly different.

This function is called by eltest4paucONE. It is listed here because the user may find it useful elsewhere.

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j \; ; \qquad \sum v_i = 1 \; , \quad \sum \nu_j = 1 \; .$$

### Value

A list containing

lambda The final tilting parameter.

u the new u vector.
v The new v vector.

"-2LLR" The -2 log empirical likelihood ratio.

iterNum The iteration number used in computing.

### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

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### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

findLnew

Finding the Lower bound of a confidence interval for theta by repeatedly testing the hypothesis for the parameter theta.

### Description

This function try to find the Lower bound of a confidence interval by repeatedly testing the hypothesis for the parameter theta until we reach the level for the "-2LLR" which return from the user supplied function fun.

### Usage

### **Arguments**

step	Search step. Must $> 0$ .
initStep	The initial step from MLE. May be used if we knew the bound is far away from MLE. This help to speed up things. Should be >=0.
fun	The function that should return "-2LLR".
MLE	The MLE of the parameter.
level	The level of the confidence. Default to 3.84 which is 95 percent confidence.
tol	The error bound for achieving the level given.
	Any additional input to be passed to fun.

#### **Details**

This function is similar to findUnew().

### Value

It returns a list containing

Low Lower bound of the confidence interval.

FstepL The error when search for Lower bound.

Lvalue The final likelihood ratio value for Lower bound. Should =~ level.

### Author(s)

Mai Zhou <maizhou@gmail.com>.

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#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

findULNEW

Finding the Upper and Lower bound of a confidence interval for theta by repeatedly testing the hypothesis for the parameter theta.

### Description

This function try to find the Upper and Lower bound of a confidence interval by repeatedly testing the hypothesis for the parameter theta until we reach the given level for the "-2LLR" which is returned from the user supplied function fun.

### Usage

### **Arguments**

step	Search step. Must $> 0$ .
initStep	The initial step from MLE. May be used if we knew the bound is far away from MLE. This help to speed up things. Should be $>=0$ .
fun	The function that should return "-2LLR".
MLE	The MLE of the parameter.
level	The level of the confidence. Default to $3.84$ which is 95 percent confidence (assume df=1).
tol	The error bound for achieving the level given.
	Any additional input to be passed to fun.

#### **Details**

This function just combines the two functions findUnew() and findLnew().

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### Value

It returns a list containing

Low	Lower bound of the confidence interval.
Up	Upper bound of the confidence interval.
FstepL	The error when search for Lower bound.
FstepU	The error when search for Upper bound.
Lvalue	The final likelihood ratio value for Lower bound. Should = level.
Uvalue	The final likelihood ratio value for Upper bound. Should = level.

### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

Finding the Upper bound of a confidence interval for theta by repeat-
edly testing the hypothesis for the parameter theta.

### Description

This function try to find the Upper bound of a confidence interval by repeatedly testing the hypothesis for the parameter theta until we reach the level for the "-2LLR" which return from the user supplied function fun.

### Usage

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### **Arguments**

step Search step. Must > 0.

initStep The initial step from MLE. May be used if we knew the bound is far away from

MLE. This help to speed up things. Should be >=0.

fun The user supplied function that should return "-2LLR".

MLE The MLE of the parameter. An approximate value should be OK.

1evel The level of the confidence. Default to 3.84 which is 95 percent confidence.

tol The error bound for achieving the level given.

... Any additional input to be passed to fun.

#### **Details**

This function is similar to findLnew().

The search is separate (for upper and lower) since Upper and Lower bound may behave differently and require different nuisance parameters inputs (in . . .).

#### Value

It returns a list containing

Up Upper bound of the confidence interval.

FstepU The error when search for Upper bound.

Uvalue The final likelihood ratio value for Upper bound. Should =~ level.

### Author(s)

Mai Zhou <maizhou@gmail.com>.

### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

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myEstPaucT	Given the x, y 2-sample data, first estimate the (1-partial)-th quantile of X sample, then estimate the pAUC(0, partial), with the plug-in estimated quantile.
	marca quamire.

### Description

This function computes the estimator using smoothed Indicator function I[x < y] which is a 3rd order polynomial.

eps is a scalar, must > 0. It is the smoothing window width for indicator function when compare x-y. epsT is a scalar, must > 0. It is the smoothing window width for estimating quantile function when compare x with tau.

### Usage

```
myEstPaucT(x, y, partial, eps=0.05, epsT=(length(x))^(-0.75))
```

### **Arguments**

X	a vector of observations, length m, for the first sample. Test-results with healthy subjects.
у	a vector of observations, length n, for the second sample. Test-results with desease subjects.
partial	The probability in pAUC(0, partial).
eps	The smoothing window width, for indicator $I[x < y]$ . Must >0.
epsT	The smoothing window width for the quantile estimation. Must >0.

### Details

This function gives the estimators that are consistant with our smoothing used in the computation of empirical likelihood. Typically the smoother for quantile should be a bit "smoother" that usual.

Known problem: when input partial is too close to 1 or 0, it will fail. When partial is equal to 1, then the pAUC reduces to AUC, which can be estimated easily, as the example below shows.

Why not also give the AUC estimator? May be it is too easy. See example below.

#### Value

```
This function returns a list, with "tau(1-partial)" $$ The estimator for the (1-partial)-th quantile of X-distribution, and <math display="block">"Pauc(\emptyset,partial)" $$ The estimated pAUC.
```

New,el2.test

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <-c(10, 209, 273, 279, 324, 391, 566, 785) x <-c(21, 38, 39, 51, 77, 185, 240, 289, 524) #### To get an estimator of the AUC (not pAUC), using our smooth3(), we do sum(smooth3(x=x, y=y))/(length(x)*length(y)) #### We should get AUC = 0.75. #### To get the estimator of the pAUC(0, 0.3) AND the 70th quantile of x sample myEstPaucT(x=x, y=y, partial=0.3) #### We should get estimator tau(1-0.3)=239.9474 and pAUC(0, 0.3)=0.1416667.
```

New.el2.test

One Iteration of Computing Paired Two Sample Constrained NPMLE.

### Description

This function computes the two sample Constrained Data NPMLE: u vectoe and v vector.

#### Usage

```
New.el2.test(u0, v0, indicmat, tol.H0, step=0.001)
```

### **Arguments**

u0 The constrained NPMLE from initial probability for sample one.
 v0 given constrained NPMLE, initial probability for sample two
 indicmat A Matrix of (smoothed) indicator values: I[x[i] < y[j]] - theta.</li>
 tol.H0 a positive number for solve constrained NPMLE tolerance.
 step a positive number. Used to approximate the dirivative of lambda function at 0.

### **Details**

The constraint is specified by matrix indicmat in

$$u\% * \% indicmat\% * \% t(v) = 0$$

The empirical likelihood we used here is defined as

$$EL = \prod_{i=1}^{m} v_i \prod_{j=1}^{n} \nu_j \; ; \quad \sum v_i = 1 \; , \quad \sum \nu_j = 1 \; .$$

quantONE 19

#### Value

A list that is the same as el2.cen.EMs() from emplik2 package. Which contains

u The new constrained NPMLE. X sample.

v The new constrained NPMLE. Y sample.

lam The lambda value.

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

quant0NE

*Smoothed quantile estimation from the given x-sample.* 

### Description

This function computes the smoothed quantile estimate, using the smoothing function smooth3. (or it can be changed easily) Otherwise it is similar to R function quantile(). Compare to quantile(x, prob, type=9).

myeps is a scalar, must > 0. It is the smoothing window width.

#### Usage

```
quantONE(x, prob, myeps=(length(x))^{-0.75})
```

### **Arguments**

x a vector of observations, length m, the sample.

prob a probabilty.

myeps The smoothing window width, must >0.

20 smooth0

#### **Details**

This function is called by myEstPaucT. It is listed here because the user may find it useful elsewhere.

Known problems: when the input prob is too close to 0 or 1 (distance less than 0.03), the computation will stop. Also, if the solution to the equation

$$\tau \mid \frac{1}{m} \sum_{i=1}^{m} smooth3vec(X_i, \tau, myeps) = prob$$

is not unique, this function only returns one of the solutions.

#### Value

The function quantONE returns a scalar that is the estimated (prob)-th quantile of X distribution.

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

#### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
#### To estimate the 70-th percentile of x distrubution:
quantONE(x=x, prob=0.7)
#### we should get 239.9474.</pre>
```

smooth0

*Non-Smoothed indicator function* 0.5I[x=y] + I[x < y].

#### **Description**

This function computes the non-smoothed Indicator function 0.5I[x=y] + I[x < y].

#### Usage

```
smooth0(x, y)
```

### **Arguments**

x a vector of observations, length m, for the first sample.

y a vector of observations, length n, for the second sample.

smooth3 21

#### **Details**

This function is used in the original AUC computation (non-smoothed). You may achieve the same result with a (data dependent) small eps in the function smooth3(x,y,eps). But this is faster and no eps to worry about. It is listed here because users may find it useful.

#### Value

smooth0() returns a matrix of dimension ncol=length(y), nrow=length(x). The entry of the matrix are values of 0.5I[x[i] = y[j]] + I[x[i] < y[j]].

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

smooth3

Smoothed indicator function I[x < y], which is the integration of the Epanechnikov kernal.

### **Description**

This function computes the smoothed Indicator function I[x < y] using a 3rd order polynomial.

If |x-y| > eps then the result is the same as the indicator function I[x < y] (either 0 or 1). For |x-y| < eps, it is a 3rd order polynomial.

eps is a scalar, must > 0. It is the smoothing window width.

### Usage

```
smooth3(x, y, eps=0.05)
```

#### **Arguments**

x a vector of observations, length m, for the first sample.

y a vector of observations, length n, for the second sample.

eps The smoothing window width, must >0.

smooth3vec

#### **Details**

This function is used in many places to replace an indicator function I[x < y]. For example, when estimating the AUC. It is listed here because users may find it useful elsewhere.

#### Value

smooth3() returns a matrix of dimension ncol=length(y), nrow=length(x). The entry of the matrix are smoothed values of I[x[i] < y[j]].

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

smooth3vec

Smoothed indicator function I[x < const], which is the integration of the Epanechnikov kernal.

### **Description**

This function computes the smoothed Indicator function I[x < const] using a 3rd order polynomial.

If |x - const| > eps then the result is the same as the indicator function I[x < const] (either 0 or 1). For |x - const| < eps, it is a 3rd order polynomial.

eps is a scalar, must > 0. It is the smoothing window width.

### Usage

```
smooth3vec(x, const, eps=0.05)
```

#### **Arguments**

x a vector of observations, length m, for the first sample.

const a single number.

eps The smoothing window width, must be >0. Ideally this should be sample size

dependent.

smooth5vec 23

#### **Details**

This function is similar to smooth3 but only compare the x vector to a single number and thus returns a vector instead of matrix. You may also use the smooth3() with a bit care, for that matter, but this vector version should be faster and save memory. It is listed here because the user may find it useful elsewhere.

We used this function to estimate the quantile from the x-sample.

#### Value

smooth3vec returns a vector of length=length(x). The entry of the vector are smoothed values of I[x[i] < const].

### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

smooth5vec

Smoothed indicator function I[x < const], which is the integration of the Quartic kernal.

#### **Description**

This function computes the smoothed Indicator function I[x < const] using a 5th order polynomial.

If |x - const| > eps then the result is the same as the indicator function I[x < const] (either 0 or 1). For |x - const| < eps, it is a 5th order polynomial.

eps is a scalar, must > 0. It is the smoothing window width.

### Usage

```
smooth5vec(x, const, eps=0.05)
```

### **Arguments**

x a vector of observations, length m, for the first sample.

const a single number.

eps The smoothing window width, must be >0. Ideally, this should be sample size

dependent.

24 smoothLvec

#### **Details**

This function is twice continuously differenciable, smoother than smooth3vec. It is listed here because the user may need extra smoothness (compare to smooth3vec) and may find it useful elsewhere.

#### Value

smooth5vec returns a vector of length=length(x). The entry of the vector are smoothed values of I[x[i] < const].

#### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

smoothLvec

Smoothed indicator function I[x < const], linear connecting the 1 and 0 on [-eps, eps].

### **Description**

This function computes the smoothed Indicator function I[x < const] using a liare.

If |x - const| > eps then the result is the same as the indicator function I[x < const] (either 0 or 1). For |x - const| < eps, it is a linear function.

eps is a scalar, must > 0. It is the smoothing window width.

#### Usage

```
smoothLvec(x, const, eps=0.05)
```

#### **Arguments**

a vector of observations, length m, for the first sample.

const a single number.

eps The smoothing window width, must be >0. Ideally this should be sample size

dependent.

smoothLvec 25

### **Details**

This function is similar to smooth3vec but only compare the x vector to a single number and thus returns a vector instead of matrix. You may also use the smooth3() with a bit care, for that matter, but this vector version should be faster and save memory. It is listed here because the user may find it useful elsewhere.

We used this function to estimate the quantile from the x-sample.

#### Value

smoothLvec returns a vector of length=length(x). The entry of the vector are smoothed values of I[x[i] < const].

### Author(s)

Mai Zhou <maizhou@gmail.com>.

#### References

Zhao, Y., Ding, X. and Zhou (2021). Confidence Intervals of AUC and pAUC by Empirical Likelihood. Tech Report. https://www.ms.uky.edu/~mai/research/eAUC1.pdf

### **Examples**

```
y <- c(10, 209, 273, 279, 324, 391, 566, 785)
x <- c(21, 38, 39, 51, 77, 185, 240, 289, 524)
```

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