Package 'L1pack'

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Title Routines for L1 Estimation

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Description L1 estimation for linear regression using Barrodale and Roberts' method <doi:10.1145/355616.361024> and the EM algorithm <doi:10.1023/A:1020759012226>. Estimation of mean and covariance matrix using the multivariate Laplace distribution, density, distribution function, quantile function and random number generation for univariate and multivariate Laplace distribution <doi:10.1080/03610929808832115>. Implementation of Naik and Plungpongpun <doi:10.1007/0-8176-4487-3_7> for the Generalized spatial median estimator is included.

Depends R(>= 3.5.0), fastmatrix

LinkingTo fastmatrix

Imports stats, grDevices, graphics

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URL https://github.com/faosorios/L1pack

NeedsCompilation yes

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Contents

`confint.lad` *Confidence intervals from* `lad` *models*

Description

Computes confidence intervals for one or more parameters in a fitted model associated to a `lad` object.

Usage

```
## S3 method for class 'lad'
confint(object, parm, level = 0.95, ...)
```

Arguments

`object`	a fitted model object.
`parm`	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
`level`	the confidence level required.
`...`	additional argument(s) for related methods.

Details

`confint` is a generic function. Confidence intervals associated to `lad` objects are asymptotic, and needs suitable `coef` and `vcov` methods to be available.

Value

A matrix (or vector) with columns giving lower and upper confidence limits for each parameter. These will be labelled as `(1-level)/2` and `1 - (1-level)/2` in % (by default 2.5% and 97.5%).

See Also

confint.glm and confint.nls in package **MASS**.

Examples

```
fm <- lad(stack.loss ~ ., data = stackloss, method = "BR")
confint(fm) # based on asymptotic normality
```

---

envelope.Laplace *QQ-plot with simulated envelopes*

Description

Constructs a normal QQ-plot using a Wilson-Hilferty transformation for the estimated Mahalanobis distances obtained from the LaplaceFit procedure.

Usage

```
envelope.Laplace(object, reps = 50, conf = 0.95, plot.it = TRUE)
```

Arguments

object	an object of class LaplaceFit representing the fitted model.
reps	number of simulated point patterns to be generated when computing the envelopes. The default number is 50.
conf	the confidence level of the envelopes required. The default is to find 95% confidence envelopes.
plot.it	if TRUE it will draw the corresponding plot, if FALSE it will only return the computed values.

Value

A list with the following components :

transformed	a vector with the z-scores obtained from the Wilson-Hilferty transformation.
envelope	a matrix with two columns corresponding to the values of the lower and upper pointwise confidence envelope.

References

Atkinson, A.C. (1985). *Plots, Transformations and Regression*. Oxford University Press, Oxford.

Osorio, F., Galea, M., Henriquez, C., Arellano-Valle, R. (2023). Addressing non-normality in multivariate analysis using the t-distribution. *AStA Advances in Statistical Analysis* **107** 785-813.

Vallejos, R., Osorio, F., Ferrer, C. (2025+). A new coefficient to measure agreement between continuous variables. Working paper.

ereturns *Excess returns for Martin Marietta and American Can companies*

Description

Data from the Martin Marietta and American Can companies collected over a period of 5 years on a monthly basis.

Usage

```
data(ereturns)
```

Format

A data frame with 60 observations on the following 4 variables.

Date the month in which the observations were collected.

am.can excess returns from the American Can company.

m.marietta excess returns from the Martin Marietta company.

CRSP an index for the excess rate returns for the New York stock exchange.

Source

Butler, R.J., McDonald, J.B., Nelson, R.D., and White, S.B. (1990). Robust and partially adaptive estimation of regression models. *The Review of Economics and Statistics* **72**, 321-327.

l1ccc *L1 concordance correlation coefficient*

Description

Calculates *L1* concordance correlation coefficient for evaluating the degree of agreement between measurements generated by two different methods.

Usage

```
l1ccc(x, data, equal.means = FALSE, boots = TRUE, nsamples = 1000, subset, na.action)
```

Arguments

x	a formula or a numeric matrix or an object that can be coerced to a numeric matrix.
data	an optional data frame (or similar: see model.frame), used only if x is a formula. By default the variables are taken from environment(formula).
equal.means	logical, should the means of the measuring devices be considered equal? In which case the restricted estimation is carried out under this assumption.
boots	logical, hould use bootstrap to approximate the variances of the *L1* estimators.
nsamples	number of bootstrap samples (default to 1000), only used if boots = TRUE.
subset	an optional expression indicating the subset of the rows of data that should be used in the fitting process.
na.action	a function that indicates what should happen when the data contain NAs.

Value

A list with class 'l1ccc' containing the following named components:

call	a list containing an image of the ccc call that produced the object.
x	data.frame used in the estimation process.
rho1	*L1* estimate of the concordance correlation coefficient.
var.rho1	approximate variance of the *L1* concordance correlation coefficient, only present if boots = TRUE.
L1	list with *L1* estimates using Laplace, and normal distributions, and *U*-statistics.
Lin	Lin's concordance correlation coefficient under the Laplace distribution.
ustat	*L1* estimation of concordance coefficient using *U*-statistics, a list containing the following elements rho1, var.rho1, ustat, and cov.
center	the estimated mean vector.
Scatter	the estimated Scatter (or Scale) matrix.
logLik	the log-likelihood at convergence.
weights	estimated weights corresponding to the Laplace distribution.
Restricted	available only if equal.means = TRUE, in which case this element corresponds to a list containing the following elements ccc, rho1, var.rho1, accuracy, precision, shifts, and information about the restrcited Fitted model.

References

King, T.S., Chinchilli, V.M. (2001). A generalized concordance correlation coefficient for continuous and categorical data. *Statistics in Medicine* **20**, 2131-2147.

King, T.S., Chinchilli, V.M. (2001). Robust estimators of the concordance correlation coefficient. *Journal of Biopharmaceutical Statistics* **11**, 83-105.

Lin, L. (1989). A concordance correlation coefficient to evaluate reproducibility. *Biometrics* **45**, 255-268.

Vallejos, R., Osorio, F., Ferrer, C. (2025+). A new coefficient to measure agreement between two continuous variables. Working paper.

Examples

```
## data from Bland and Altman (1986). The Lancet 327, 307-310.
x <- list(Large = c(494,395,516,434,476,557,413,442,650,433,
          417,656,267,478,178,423,427),
          Mini  = c(512,430,520,428,500,600,364,380,658,445,
          432,626,260,477,259,350,451))
x <- as.data.frame(x)

plot(Mini ~ Large, data = x, xlim = c(100,800), ylim = c(100,800),
     xlab = "PERF by Large meter", ylab = "PERF by Mini meter")
abline(c(0,1), col = "gray", lwd = 2)

## estimating L1 concordance coefficient
z <- l1ccc(~ Mini + Large, data = x, boots = FALSE)
z
## output:
# Call:
# l1ccc(x = ~ Mini + Large, data = x, boots = FALSE)
#
# L1 coefficients using:
# Laplace  Gaussian  U-statistic
#  0.7456    0.7607       0.7642
#
# Lin's coefficients:
# estimate  accuracy precision
#   0.9395    0.9974    0.9419
```

`l1fit` *Minimum absolute residual (L1) regression*

Description

Performs an L1 regression on a matrix of explanatory variables and a vector of responses.

Usage

```
l1fit(x, y, intercept = TRUE, tolerance = 1e-07, print.it = TRUE)
```

Arguments

`x`	vector or matrix of explanatory variables. Each row corresponds to an observation and each column to a variable. The number of rows of `x` should equal the number of data values in `y`, and there should be fewer columns than rows. Missing values are not allowed.
`y`	numeric vector containing the response. Missing values are not allowed.
`intercept`	logical flag. If `TRUE`, an intercept term is included in the regression model.
`tolerance`	numerical value used to test for singularity in the regression.
`print.it`	logical flag. If `TRUE`, then warnings about non-unique solutions and rank deficiency are given.

Details

The Barrodale-Roberts algorithm, which is a specialized linear programming algorithm, is used.

Value

list defining the regression (compare with function `lsfit`).

`coefficients`	vector of coefficients.
`residuals`	residuals from the fit.
`message`	character strings stating whether a non-unique solution is possible, or if the `x` matrix was found to be rank deficient.

References

Barrodale, I., Roberts, F.D.K. (1973). An improved algorithm for discrete L1 linear approximations. *SIAM Journal of Numerical Analysis* **10**, 839-848.

Barrodale, I., Roberts, F.D.K. (1974). Solution of an overdetermined system of equations in the L1 norm. *Communications of the ACM* **17**, 319-320.

Bloomfield, P., Steiger, W.L. (1983). *Least Absolute Deviations: Theory, Applications, and Algorithms.* Birkhauser, Boston, Mass.

Examples

```
l1fit(stack.x, stack.loss)
```

`lad` *Least absolute deviations regression*

Description

This function is used to fit linear models considering Laplace errors.

Usage

```
lad(formula, data, subset, na.action, method = "BR", tol = 1e-7, maxiter = 200,
  x = FALSE, y = FALSE, contrasts = NULL)
```

Arguments

`formula`	an object of class `"formula"`: a symbolic description of the model to be fitted.
`data`	an optional data frame containing the variables in the model. If not found in `data`, the variables are taken from `environment(formula)`, typically the environment from which `lad` is called.
`subset`	an optional expression indicating the subset of the rows of data that should be used in the fit.

na.action	a function that indicates what should happen when the data contain NAs.
method	character string specifying the fitting method to be used; the options are "BR" Barrodale and Roberts' method (the default) and "EM" for an EM algorithm using IRLS.
tol	the relative tolerance for the iterative algorithm. Default is tol = 1e-7.
maxiter	The maximum number of iterations for the EM method. Default to 200.
x, y	logicals. If TRUE the corresponding components of the fit (the model matrix, the response) are returned.
contrasts	an optional list. See the contrasts.arg of model.matrix.default.

Value

An object of class lad representing the linear model fit. Generic function print, show the results of the fit.

The functions print and summary are used to obtain and print a summary of the results. The generic accessor functions coefficients, fitted.values and residuals extract various useful features of the value returned by lad.

Author(s)

The design was inspired by the R function lm.

References

Barrodale, I., Roberts, F.D.K. (1974). Solution of an overdetermined system of equations in the L1 norm. *Communications of the ACM* **17**, 319-320.

Phillips, R.F. (2002). Least absolute deviations estimation via the EM algorithm. *Statistics and Computing* **12**, 281-285.

Examples

```
fm <- lad(stack.loss ~ ., data = stackloss, method = "BR")
summary(fm)

data(ereturns)
fm <- lad(m.marietta ~ CRSP, data = ereturns, method = "EM")
summary(fm)
# basic observations
fm$basic
```

lad.fit *Fitter functions for least absolute deviation (LAD) regression*

Description

This function is a *switcher* among various numerical fitting functions (lad.fit.BR, and lad.fit.EM). The argument method does the switching: "BR" for lad.fit.BR, etc. This should usually *not* be used directly unless by experienced users.

Usage

```
lad.fit(x, y, method = "BR", tol = 1e-7, maxiter = 200)
```

Arguments

x	design matrix of dimension $n \times p$.
y	vector of observations of length n.
method	currently, methods "BR" (default), and "EM" are supported.
tol	the relative tolerance for the iterative algorithm. Default is tol = 1e-7.
maxiter	The maximum number of iterations for the EM method. Default to 200.

Value

a list with components:

coefficients	a named vector of coefficients.
scale	final scale estimate of the random error.
residuals	the residuals, that is response minus fitted values.
fitted.values	the fitted values.
SAD	the sum of absolute deviations.
weights	estimated EM weights.
basic	basic observations, that is observations with zero residuals.
logLik	the log-likelihood at convergence.

See Also

lad.fit.BR, lad.fit.EM.

Examples

```
x <- cbind(1, stack.x)
fm <- lad.fit(x, stack.loss, method = "BR")
fm
```

lad.fit-methods *Fit a least absolute deviation (LAD) regression model*

Description

Fits a linear model using LAD methods, returning the bare minimum computations.

Usage

```
lad.fit.BR(x, y, tol = 1e-7)
lad.fit.EM(x, y, tol = 1e-7, maxiter = 200)
```

Arguments

x, y	numeric vectors or matrices for the predictors and the response in a linear model. Typically, but not necessarily, x will be constructed by one of the fitting functions.
tol	the relative tolerance for the iterative algorithm. Default is tol = 1e-7.
maxiter	The maximum number of iterations for the EM method. Default to 200.

Value

The bare bones of a lad object: the coefficients, residuals, fitted values, and some information used by summary.lad.

See Also

lad, lad.fit, lm

Examples

```
x <- cbind(1, stack.x)
z <- lad.fit.BR(x, stack.loss)
z
```

Laplace *The symmetric Laplace distribution*

Description

Density, distribution function, quantile function and random generation for the Laplace distribution with location parameter location and scale parameter scale.

Usage

```
dlaplace(x, location = 0, scale = 1, log = FALSE)
plaplace(q, location = 0, scale = 1, lower.tail = TRUE, log.p = FALSE)
qlaplace(p, location = 0, scale = 1, lower.tail = TRUE, log.p = FALSE)
rlaplace(n, location = 0, scale = 1)
```

Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations.
location	location parameter μ, which is the mean.
scale	scale parameter ϕ. Scale must be positive.
log, log.p	logical; if TRUE, probabilities p are given as $\log(p)$.
lower.tail	logical; if TRUE (default), probabilities are $P(X \leq x)$, otherwise, $P(X > x)$.

Details

If location or scale are not specified, they assume the default values of 0 and 1 respectively.

The Laplace distribution with location μ and scale ϕ has density

$$f(x) = \frac{1}{\sqrt{2}\phi} \exp(-\sqrt{2}|x-\mu|/\phi),$$

where $-\infty < y < \infty$, $-\infty < \mu < \infty$ and $\phi > 0$. The mean is μ and the variance is ϕ^2.

The cumulative distribution function, assumes the form

$$F(x) = \begin{cases} \frac{1}{2}\exp(\sqrt{2}(x-\mu)/\phi) & x < \mu, \\ 1 - \frac{1}{2}\exp(-\sqrt{2}(x-\mu)/\phi) & x \geq \mu. \end{cases}$$

The quantile function, is given by

$$F^{-1}(p) = \begin{cases} \mu + \frac{\phi}{\sqrt{2}}\log(2p) & p < 0.5, \\ \mu - \frac{\phi}{\sqrt{2}}\log(2(1-p)) & p \geq 0.5. \end{cases}$$

Value

dlaplace, plaplace, and qlaplace are respectively the density, distribution function and quantile function of the Laplace distribution. rlaplace generates random deviates drawn from the Laplace distribution, the length of the result is determined by n.

Author(s)

Felipe Osorio and Tymoteusz Wolodzko

References

Kotz, S., Kozubowski, T.J., Podgorski, K. (2001). *The Laplace Distributions and Generalizations*. Birkhauser, Boston.

Krishnamoorthy, K. (2006). *Handbook of Statistical Distributions with Applications*, 2nd Ed. Chapman & Hall, Boca Raton.

See Also

Distributions for other standard distributions and `rmLaplace` for the random generation from the multivariate Laplace distribution.

Examples

```
x <- rlaplace(1000)
## QQ-plot for Laplace data against true theoretical distribution:
qqplot(qlaplace(ppoints(1000)), x, main = "Laplace QQ-plot",
  xlab = "Theoretical quantiles", ylab = "Sample quantiles")
abline(c(0,1), col = "red", lwd = 2)
```

LaplaceFit *Estimation of location and scatter using the multivariate Laplace distribution*

Description

Estimates the location vector and scatter matrix assuming the data came from a multivariate Laplace distribution.

Usage

```
LaplaceFit(x, data, subset, na.action, tol = 1e-6, maxiter = 200)
```

Arguments

`x`	a formula or a numeric matrix or an object that can be coerced to a numeric matrix.
`data`	an optional data frame (or similar: see `model.frame`), used only if `x` is a formula. By default the variables are taken from `environment(formula)`.
`subset`	an optional expression indicating the subset of the rows of data that should be used in the fitting process.
`na.action`	a function that indicates what should happen when the data contain NAs.
`tol`	the relative tolerance in the iterative algorithm.
`maxiter`	maximum number of iterations. The default is 200.

Value

A list with class 'LaplaceFit' containing the following components:

call	a list containing an image of the LaplaceFit call that produced the object.
center	final estimate of the location vector.
Scatter	final estimate of the scale matrix.
logLik	the log-likelihood at convergence.
numIter	the number of iterations used in the iterative algorithm.
weights	estimated weights corresponding to the Laplace distribution.
distances	estimated squared Mahalanobis distances.

Generic function print show the results of the fit.

References

Yavuz, F.G., Arslan, O. (2018). Linear mixed model with Laplace distribution (LLMM). *Statistical Papers* **59**, 271-289.

See Also

cov

Examples

```
fit <- LaplaceFit(stack.x)
fit

# covariance matrix
p <- fit$dims[2]
Sigma <- (4 * (p + 1)) * fit$Scatter
Sigma
```

---

mLaplace *Multivariate Laplace distribution*

Description

These functions provide the density and random number generation from the multivariate Laplace distribution.

Usage

```
dmLaplace(x, center = rep(0, nrow(Scatter)), Scatter = diag(length(center)), log = FALSE)
rmLaplace(n = 1, center = rep(0, nrow(Scatter)), Scatter = diag(length(center)))
```

Arguments

x	vector or matrix of data.
n	the number of samples requested.
center	a $k \times 1$ vector giving the locations.
Scatter	a $k \times k$ positive-definite dispersion matrix.
log	logical; if TRUE, the logarithm of the density function is returned.

Details

The multivariate Laplace distribution is a multidimensional extension of the univariate symmetric Laplace distribution. There are multiple forms of the multivariate Laplace distribution. Here, a particular case of the multivarite power exponential distribution introduced by Gomez et al. (1998) is considered.

The multivariate Laplace distribution with location $\boldsymbol{\mu}$ = center and $\boldsymbol{\Sigma}$ = Scatter has density

$$f(\boldsymbol{x}) = \frac{\Gamma(k/2)}{\pi^{k/2}\Gamma(k)2^{k+1}}|\boldsymbol{\Sigma}|^{-1/2}\exp\left\{-\frac{1}{2}[(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})]^{1/2}\right\}.$$

The function rmLaplace is an interface to C routines, which make calls to subroutines from LAPACK. The matrix decomposition is internally done using the Cholesky decomposition. If Scatter is not non-negative definite then there will be a warning message.

Value

If x is a matrix with n rows, then dmLaplace returns a $n \times 1$ vector considering each row of x as a copy from the multivariate Laplace.

If n = 1, then rmLaplace returns a vector of the same length as center, otherwise a matrix of n rows of random vectors.

References

Fang, K.T., Kotz, S., Ng, K.W. (1990). *Symmetric Multivariate and Related Distributions*. Chapman & Hall, London.

Gomez, E., Gomez-Villegas, M.A., Marin, J.M. (1998). A multivariate generalization of the power exponential family of distributions. *Communications in Statistics - Theory and Methods* **27**, 589-600.

Examples

```
# dispersion parameters
Scatter <- matrix(c(1,.5,.5,1), ncol = 2)
Scatter

# generate the sample
y <- rmLaplace(n = 2000, Scatter = Scatter)

# scatterplot of a random bivariate Laplace sample with center
# vector zero and scale matrix 'Scatter'
```

```
par(pty = "s")
plot(y, xlab = "", ylab = "")
title("bivariate Laplace sample", font.main = 1)
```

`simulate.lad` *Simulate responses from* `lad` *models*

Description

Simulate one or more responses from the distribution corresponding to a fitted `lad` object.

Usage

```
## S3 method for class 'lad'
simulate(object, nsim = 1, seed = NULL, ...)
```

Arguments

`object`	an object representing a fitted model.
`nsim`	number of response vectors to simulate. Defaults to 1.
`seed`	an object specifying if and how the random number generator should be initialized ('seeded'). For the "lad" method, either `NULL` or an integer that will be used in a call to `set.seed` before simulating the response vectors. If set, the value is saved as the `"seed"` attribute of the returned value. The default, `NULL` will not change the random generator state, and return `.Random.seed` as the `"seed"` attribute, see 'Value'.
`...`	additional optional arguments.

Value

For the `"lad"` method, the result is a data frame with an attribute `"seed"`. If argument `seed` is `NULL`, the attribute is the value of `.Random.seed` before the simulation was started.

Author(s)

Tymoteusz Wolodzko and Felipe Osorio

Examples

```
fm <- lad(stack.loss ~ ., data = stackloss)
sm <- simulate(fm, nsim = 4)
```

`spatial.median` *Computation of the generalized spatial median*

Description

Computation of the generalized spatial median estimator as defined by Rao (1988).

Usage

```
spatial.median(x, data, subset, na.action, tol = 1e-6, maxiter = 200)
```

Arguments

`x`	a formula or a numeric matrix or an object that can be coerced to a numeric matrix.
`data`	an optional data frame (or similar: see `model.frame`), used only if `x` is a formula. By default the variables are taken from `environment(formula)`.
`subset`	an optional expression indicating the subset of the rows of data that should be used in the fitting process.
`na.action`	a function that indicates what should happen when the data contain NAs.
`tol`	the relative tolerance in the iterative algorithm.
`maxiter`	maximum number of iterations. The default is 200.

Details

An interesting fact is that the generalized spatial median estimator proposed by Rao (1988) is the maximum likelihood estimator under the Kotz-type distribution discussed by Naik and Plungpongpun (2006). The generalized spatial median estimators are defined as $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$ which minimize

$$\frac{n}{2}\log|\boldsymbol{\Sigma}| + \sum_{i=1}^{n}\sqrt{(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})},$$

simultaneously with respect to $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$.

The function `spatial.median` follows the iterative reweighting algorithm of Naik and Plungpongpun (2006).

Value

A list with class `'spatial.median'` containing the following components:

`call`	a list containing an image of the `spatial.median` call that produced the object.
`median`	final estimate of the location vector.
`Scatter`	final estimate of the scale matrix.
`logLik`	the log-likelihood at convergence.
`numIter`	the number of iterations used in the iterative algorithm.

`innerIter`	the total number of iterations used in the **inner** iterative algorithm.
`weights`	estimated weights corresponding to the Kotz distribution.
`distances`	estimated squared Mahalanobis distances.

Generic function `print` show the results of the fit.

References

Naik, D.N., Plungpongpun, K. (2006). A Kotz-type distribution for multivariate statistical inference. In: Balakrishnan, N., Sarabia, J.M., Castillo, E. (Eds) *Advances in Distribution Theory, Order Statistics, and Inference*. Birkhauser Boston, pp. 111-124.

Rao, C.R. (1988). Methodology based on the L1-norm in statistical inference. *Sankhya, Series A* **50**, 289-313.

See Also

`cov`, `LaplaceFit`

Examples

```
z <- spatial.median(stack.x)
z
```

`vcov.lad` *Calculate variance-covariance matrix from* `lad` *models*

Description

Returns the variance-covariance matrix of the main parameters of a fitted model for `lad` objects. The "main" parameters of model correspond to those returned by `coef`, and typically do not contain the nuisance scale parameter.

Usage

```
## S3 method for class 'lad'
vcov(object, ...)
```

Arguments

`object`	an object representing a fitted model.
`...`	additional arguments for method functions.

Value

A matrix of the estimated covariances between the parameter estimates in the linear regression model. This should have row and column names corresponding to the parameter names given by the `coef` method.

`WH.Laplace` *Wilson-Hilferty transformation*

Description

Returns the Wilson-Hilferty transformation for multivariate Laplace deviates.

Usage

```
WH.Laplace(x, center, Scatter)
```

Arguments

`x`	object of class `'LaplaceFit'` from which is extracted the estimated Mahalanobis distances of the fitted model. Also `x` can be a vector or matrix of data with, say, p columns.
`center`	mean vector of the distribution or data vector of length p. Not required if `x` have class `'LaplaceFit'`.
`Scatter`	Scatter matrix (p by p) of the distribution. Not required if `x` have class `'LaplaceFit'`.

Details

Let $T = D/(2p)$ be a Gamma distributed random variable, where D^2 denotes the squared Mahalanobis distance defined as

$$D^2 = (\boldsymbol{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu}).$$

Thus, the Wilson-Hilferty transformation is given by

$$z = \frac{T^{1/3} - (1 - \frac{1}{9p})}{(\frac{1}{9p})^{1/2}}$$

and z is approximately distributed as a standard normal distribution. This is useful, for instance, in the construction of QQ-plots.

References

Osorio, F., Galea, M., Henriquez, C., Arellano-Valle, R. (2023). Addressing non-normality in multivariate analysis using the *t*-distribution. *AStA Advances in Statistical Analysis* **107**, 785-813.

Terrell, G.R. (2003). The Wilson-Hilferty transformation is locally saddlepoint. *Biometrika* **90**, 445-453.

Wilson, E.B., Hilferty, M.M. (1931). The distribution of chi-square. *Proceedings of the National Academy of Sciences of the United States of America* **17**, 684-688.

Examples

```
Scatter <- matrix(c(1,.5,.5,1), ncol = 2)
Scatter

# generate the sample
y <- rmLaplace(n = 500, Scatter = Scatter)
fit <- LaplaceFit(y)
z <- WH.Laplace(fit)
par(pty = "s")
qqnorm(z, main = "Transformed distances QQ-plot")
abline(c(0,1), col = "red", lwd = 2)
```

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