

# Package ‘skellam’

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**Title** Densities and Sampling for the Skellam Distribution

**Description** Functions for the Skellam distribution, including: density (pmf), cdf, quantiles and regression.

**URL** <https://github.com/monty-se/skellam>

**License** GPL (>= 2)

**Imports** stats

**Suggests** knitr, rmarkdown

**VignetteBuilder** knitr

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**Author** Jerry W. Lewis [aut],  
Patrick E. Brown [aut],  
Michail Tsagris [aut],  
Montasser Ghachem [cre]

**Maintainer** Montasser Ghachem <mg@pinstimation.com>

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 skellam

*The Skellam Distribution*


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### Description

Density, distribution function, quantile function, and random generation for the Skellam distribution.

### Usage

```
dskellam(x, lambda1, lambda2 = lambda1, log = FALSE)
```

```
dskellam.sp(x, lambda1, lambda2 = lambda1, log = FALSE)
```

```
pskellam(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

```
pskellam.sp(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

```
qskellam(p, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

```
rskellam(n, lambda1, lambda2 = lambda1)
```

### Arguments

x, q	For functions dskellam, dskellam.sp, and pskellam.sp: a numeric vector of quantiles.
lambda1, lambda2	Numeric vectors of (non-negative) means; lambda2 defaults to lambda1 if not provided.
log, log.p	Logical; if TRUE, returns the logarithm of the computed value.
lower.tail	Logical; if TRUE (default), returns $P(X \leq x)$ ; otherwise, returns $P(X > x)$ .
p	For qskellam: a numeric vector of probabilities.
n	For rskellam: a non-negative integer specifying the number of observations.

### Details

The Skellam distribution describes the difference between two independent Poisson random variables. This documentation covers:

#### Density:

```
dskellam(x, lambda1, lambda2 = lambda1, log = FALSE)
```

#### Distribution Function:

```
pskellam(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

### Quantile Function:

```
qskellam(p, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

### Random Generation:

```
rskellam(n, lambda1, lambda2 = lambda1)
```

### Saddlepoint Approximations:

```
dskellam.sp(x, lambda1, lambda2 = lambda1, log = FALSE)
```

```
pskellam.sp(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

If  $Y_1$  and  $Y_2$  are Poisson variables with means  $\mu_1$  and  $\mu_2$  and correlation  $\rho$ , then  $X = Y_1 - Y_2$  is Skellam with parameters:

$$\lambda_1 = \mu_1 - \rho\sqrt{\mu_1\mu_2}$$

$$\lambda_2 = \mu_2 - \rho\sqrt{\mu_1\mu_2}$$

The density is given by:

$$I(2\sqrt{\lambda_1\lambda_2}, |x|)(\lambda_1/\lambda_2)^{x/2} \exp(-\lambda_1 - \lambda_2)$$

where  $I(y, \nu)$  is the modified Bessel function of the first kind.

### Value

- `dskellam` returns the (log) density.
- `pskellam` returns the (log) cumulative distribution function.
- `qskellam` returns the quantile function.
- `rskellam` generates random deviates.

Invalid lambda values will return NaN with a warning.

### Note

The **VGAM** package also provides Skellam functions. This implementation offers a broader working range, correct handling when one rate parameter is zero, enhanced argument checking, and improved accuracy for  $x < 0$  (in R versions prior to 2.9). Use `skellam::dskellam` or `VGAM::dskellam` to specify which implementation to use.

## References

- Butler, R. (2007) *Saddlepoint Approximations with Applications*, Cambridge University Press.
- Johnson, N. L. (1959) On an extension of the connection between Poisson and  $\chi^2$  distributions. *Biometrika* 46, 352-362.
- Johnson, N. L., Kotz, S., & Kemp, A. W. (1993) *Univariate Discrete Distributions*, 2nd ed., John Wiley and Sons.
- Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates. *Journal of the Royal Statistical Society, Series A* 109(3), 296.
- Strackee, J. & van der Gon, J. J. D. (1962) The frequency distribution of the difference between two Poisson variates. *Statistica Neerlandica* 16(1), 17-23.
- Wikipedia: [https://en.wikipedia.org/wiki/Skellam\\_distribution](https://en.wikipedia.org/wiki/Skellam_distribution)

## Examples

```
# Compare with Poisson when one lambda = 0
dskellam(0:10, 5, 0)
dpois(0:10, 5)

# Both lambdas non-zero
dskellam(c(-1,1), c(12,10), c(10,12))
pskellam(c(-1,0), c(12,10), c(10,12))

# Quantile function
qskellam(c(0.05, 0.95), 3, 4)

# Random generation
rskellam(10, 8.5, 10.25)
```

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 skellam.mle

*Maximum Likelihood Estimation for the Skellam Distribution*


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## Description

Estimates the parameters of a Skellam distribution using maximum likelihood.

## Usage

```
skellam.mle(x)
```

## Arguments

x                    A vector of integers (positive or negative).

## Details

Instead of having to maximize the log-likelihood with respect to both parameters ( $\lambda_1$  and  $\lambda_2$ ), the function maximizes with respect to  $\lambda_2$  while setting  $\lambda_1 = \lambda_2 + \bar{x}$ . This approach improves computational efficiency. The optimization is performed using `nlm` as it proved faster than `optimise`.

## Value

A list with components:

**iters** Number of iterations required by `nlm`.

**loglik** Maximized log-likelihood value.

**param** Estimated parameters ( $\hat{\lambda}_1, \hat{\lambda}_2$ ).

## Author(s)

Michail Tsagris

## References

- Butler, R. (2007) *Saddlepoint Approximations with Applications*, Cambridge University Press.
- Johnson, N. L. (1959) On an extension of the connection between Poisson and  $\chi^2$  distributions. *Biometrika* **46**, 352-362.
- Johnson, N. L.; Kotz, S.; Kemp, A. W. (1993) *Univariate Discrete Distributions*, 2nd ed., John Wiley and Sons.
- Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates belonging to different populations. *Journal of the Royal Statistical Society, Series A* **109**(3), 296.
- Strackee, J.; van der Gon, J. J. D. (1962) The frequency distribution of the difference between two Poisson variates. *Statistica Neerlandica* **16**(1), 17-23.
- Abdulhamid, A. A.; Maha, A. O. (2010) On The Poisson Difference Distribution Inference and Applications. *Bulletin of the Malaysian Mathematical Sciences Society* **33**(1), 17-45.
- Wikipedia: Skellam distribution [https://en.wikipedia.org/wiki/Skellam\\_distribution](https://en.wikipedia.org/wiki/Skellam_distribution)

## Examples

```
# Basic example
x1 <- rpois(1000, 10)
x2 <- rpois(1000, 6)
x <- x1 - x2
skellam.mle(x)

# Larger sample size
x1 <- rpois(10000, 10)
x2 <- rpois(10000, 6)
x <- x1 - x2
skellam.mle(x)
```

---

`skellam.reg`*Skellam Regression*

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### Description

Fits a regression model assuming a Skellam distribution for the response variable.

### Usage

```
skellam.reg(y, x)
```

### Arguments

<code>y</code>	A vector of integers (positive or negative)
<code>x</code>	A matrix, vector or data.frame of covariates

### Details

The function uses an exponential link function to ensure positive values for both rate parameters ( $\lambda_1$  and  $\lambda_2$ ). Optimization is performed using `nlm`.

### Value

A list with components:

**loglik** Maximized log-likelihood value

**param1** Matrix for  $\lambda_1$  parameters:

- Column 1: Estimated coefficients
- Column 2: Standard errors
- Column 3: t-values (coef/se)
- Column 4: p-values (Wald test)

**param2** Matrix for  $\lambda_2$  parameters (same structure as param1)

### Author(s)

Michail Tsagris

### References

- Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates belonging to different populations. *Journal of the Royal Statistical Society, Series A* **109**(3), 296.
- Strackee, J.; van der Gon, J. J. D. (1962) The frequency distribution of the difference between two Poisson variates. *Statistica Neerlandica* **16**(1), 17-23.
- Karlis D. and Ntzoufras I. (2009) *Analysis of sports data using bivariate Poisson models*. IMA Conference Presentation. [http://www2.stat-athens.aueb.gr/~jbn/papers/files/20\\_Karlis\\_Ntzoufras\\_2009\\_IMA\\_presentation\\_handouts\\_v01.pdf](http://www2.stat-athens.aueb.gr/~jbn/papers/files/20_Karlis_Ntzoufras_2009_IMA_presentation_handouts_v01.pdf)

**Examples**

```
set.seed(0)
x <- rnorm(100)
y1 <- rpois(100, exp(1 + 1 * x))
y2 <- rpois(100, exp(-1 + 1 * x))
y <- y2 - y1
skellam.reg(y, x)
```

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