Differential analysis of count data – the DESeq2 package

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Abstract

A basic task in the analysis of count data from RNA-Seq is the detection of differentially expressed genes. The count data are presented as a table which reports, for each sample, the number of sequence fragments that have been assigned to each gene. Analogous data also arise for other assay types, including comparative ChIP-Seq, HiC, shRNA screening, mass spectrometry. An important analysis question is the quantification and statistical inference of systematic changes between conditions, as compared to within-condition variability. The package *DESeq2* provides methods to test for differential expression by use of negative binomial generalized linear models; the estimates of dispersion and logarithmic fold changes incorporate data-driven prior distributions¹. This vignette explains the use of the package and demonstrates typical workflows. An RNA-Seq workflow on the Bioconductor website: http://www.bioconductor.org/help/workflows/rnaseqGene/ (formerly the Beginner's Guide PDF), covers similar material to this vignette but at a slower pace, including the generation of count matrices from FASTQ files.

DESeq2 version: 1.6.3

If you use *DESeq2* in published research, please cite:

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¹Other *Bioconductor* packages with similar aims are *edgeR*, *baySeq*, *DSS* and *limma*.

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1 Standard workflow

1.1 Quick start

Here we show the most basic steps for a differential expression analysis. These steps imply you have a *SummarizedExperiment* object se with a column condition in colData(se).

```
dds <- DESeqDataSet(se = se, design = ~ condition)
dds <- DESeq(dds)
res <- results(dds)</pre>
```

1.2 Input data

1.2.1 Why raw counts?

As input, the *DESeq2* package expects count data as obtained, e.g., from RNA-Seq or another highthroughput sequencing experiment, in the form of a matrix of integer values. The value in the *i*-th row and the *j*-th column of the matrix tells how many reads have been mapped to gene *i* in sample *j*. Analogously, for other types of assays, the rows of the matrix might correspond e.g. to binding regions (with ChIP-Seq) or peptide sequences (with quantitative mass spectrometry).

The count values must be raw counts of sequencing reads. This is important for *DESeq2*'s statistical model [1] to hold, as only the actual counts allow assessing the measurement precision correctly. Hence, please do not supply other quantities, such as (rounded) normalized counts, or counts of covered base pairs – this will only lead to nonsensical results.

1.2.2 SummarizedExperiment input

The class used by the *DESeq2* package to store the read counts is *DESeqDataSet* which extends the *SummarizedExperiment* class of the *GenomicRanges* package. This facilitates preparation steps and also downstream exploration of results. For counting aligned reads in genes, the summarizeOverlaps function of *GenomicAlignments* with mode="Union" is encouraged, resulting in a *SummarizedExperiment* object.

An example of the steps to produce a *SummarizedExperiment* can be found in an RNA-Seq workflow on the Bioconductor website: http://www.bioconductor.org/help/workflows/rnaseqGene/ and in the vignette for the data package *airway*. Here we load the *SummarizedExperiment* from that package in order to build a *DESeqDataSet*.

```
library("airway")
data("airway")
se <- airway</pre>
```

A DESeqDataSet object must have an associated design formula. The design formula expresses the variables which will be used in modeling. The formula should be a tilde (\sim) followed by the variables with plus signs between them (it will be coerced into an *formula* if it is not already). An intercept is included, representing the base mean of counts. The design can be changed later, however then all differential analysis steps should be repeated, as the design formula is used to estimate the dispersions and to estimate the log2 fold changes of the model. The constructor function below shows the generation of a DESeqDataSet from a SummarizedExperiment se.

Note: In order to benefit from the default settings of the package, you should put the variable of interest at the end of the formula and make sure the control level is the first level.

```
library("DESeq2")
ddsSE <- DESeqDataSet(se, design = ~ cell + dex)
ddsSE
## class: DESeqDataSet
## dim: 64102 8
## exptData(1): ''
## assays(1): counts
## rownames(64102): ENSG000000003 ENSG000000005 ... LRG_98 LRG_99
## rowData metadata column names(0):
## colnames(8): SRR1039508 SRR1039509 ... SRR1039520 SRR1039521
## colData names(9): SampleName cell ... Sample BioSample</pre>
```

1.2.3 Count matrix input

Alternatively, the function DESeqDataSetFromMatrix can be used if you already have a matrix of read counts prepared. For this function you should provide the counts matrix, the column information as a *DataFrame* or *data.frame* and the design formula. First, we load the pasillaGenes data object, in order to extract a count matrix and phenotypic data.

```
library("pasilla")
library("Biobase")
data("pasillaGenes")
countData <- counts(pasillaGenes)
colData <- pData(pasillaGenes)[,c("condition","type")]</pre>
```

Now that we have a matrix of counts and the column information, we can construct a DESeqDataSet:

dds

```
## class: DESeqDataSet
## dim: 14470 7
## exptData(0):
```

```
## assays(1): counts
## rownames(14470): FBgn0000003 FBgn0000008 ... FBgn0261574 FBgn0261575
## rowData metadata column names(0):
## colnames(7): treated1fb treated2fb ... untreated3fb untreated4fb
## colData names(2): condition type
```

If you have additional feature data, it can be added to the *DESeqDataSet* by adding to the metadata columns of a newly constructed object. (Here we add redundant data for demonstration, as the gene names are already the rownames of the dds.)

```
featureData <- data.frame(gene=rownames(pasillaGenes))</pre>
(mcols(dds) <- DataFrame(mcols(dds), featureData))</pre>
## DataFrame with 14470 rows and 1 column
##
                gene
##
            <factor>
## 1
         FBgn000003
## 2
         FBgn000008
         FBgn0000014
## 3
         FBgn0000015
## 4
         FBgn0000017
## 5
## ...
                  . . .
## 14466 FBgn0261571
## 14467 FBgn0261572
## 14468 FBgn0261573
## 14469 FBgn0261574
## 14470 FBgn0261575
```

1.2.4 HTSeq input

You can use the function DESeqDataSetFromHTSeqCount if you have htseq-count from the *HTSeq* python package². For an example of using the python scripts, see the *pasilla* data package. First you will want to specify a variable which points to the directory in which the *HTSeq* output files are located.

```
directory <- "/path/to/your/files/"</pre>
```

However, for demonstration purposes only, the following line of code points to the directory for the demo *HTSeq* output files packages for the *pasilla* package.

directory <- system.file("extdata", package="pasilla", mustWork=TRUE)</pre>

We specify which files to read in using list.files, and select those files which contain the string "treated" using grep. The sub function is used to chop up the sample filename to obtain the condition status, or you might alternatively read in a phenotypic table using read.table.

²available from http://www-huber.embl.de/users/anders/HTSeq, described in [2]

```
sampleFiles <- grep("treated",list.files(directory),value=TRUE)</pre>
sampleCondition <- sub("(.*treated).*","\\1",sampleFiles)</pre>
sampleTable <- data.frame(sampleName = sampleFiles,</pre>
                           fileName = sampleFiles,
                           condition = sampleCondition)
ddsHTSeq <- DESeqDataSetFromHTSeqCount(sampleTable = sampleTable,</pre>
                                         directory = directory,
                                         design= ~ condition)
ddsHTSeq
## class: DESeqDataSet
## dim: 70463 7
## exptData(0):
## assays(1): counts
## rownames(70463): FBgn0000003:001 FBgn0000008:001 ... FBgn0261575:001
     FBgn0261575:002
##
## rowData metadata column names(0):
## colnames(7): treated1fb.txt treated2fb.txt ... untreated3fb.txt
##
     untreated4fb.txt
## colData names(1): condition
```

1.2.5 Note on factor levels

In the three examples above, we applied the function factor to the column of interest in colData, supplying a character vector of levels. It is important to supply levels (otherwise the levels are chosen in alphabetical order) and to put the "control" or "untreated" level as the first element ("base level"), so that the log2 fold changes produced by default will be the expected comparison against the base level. An *R* function for easily changing the base level is relevel. An example of setting the base level of a factor with relevel is:

```
dds$condition <- relevel(dds$condition, "untreated")</pre>
```

In addition, when subsetting the columns of a *DESeqDataSet*, i.e., when removing certain samples from the analysis, it is possible that all the samples for one or more levels of a variable in the design formula are removed. In this case, the droplevels function can be used to remove those levels which do not have samples in the current *DESeqDataSet*:

```
dds$condition <- droplevels(dds$condition)</pre>
```

1.2.6 Collapsing technical replicates

DESeq2 provides a function collapseReplicates which can assist in combining the counts from technical replicates into single columns. See the manual page for an example of the use of collapseReplicates.

1.2.7 About the pasilla dataset

We continue with the *pasilla* data constructed from the count matrix method above. This data set is from an experiment on *Drosophila melanogaster* cell cultures and investigated the effect of RNAi knock-down of the splicing factor *pasilla* [3]. The detailed transcript of the production of the *pasilla* data is provided in the vignette of the data package *pasilla*.

1.3 Differential expression analysis

The standard differential expression analysis steps are wrapped into a single function, DESeq. The steps of this function are described in Section 4.1 and in the manual page for ?DESeq. The individual sub-functions which are called by DESeq are still available, described in Section 3.1.

Results tables are generated using the function results, which extracts a results table with $\log 2$ fold changes, p values and adjusted p values. With no arguments to results, the results will be for the last variable in the design formula, and if this is a factor, the comparison will be the last level of this variable over the first level.

dds <- DESeq(dds)
res <- results(dds)</pre>

These steps should take less than 30 seconds for most analyses. For experiments with many samples (e.g. 100 samples), one can take advantage of parallelized computation. Both of the above functions have an argument parallel which if set to TRUE can be used to distribute computation across cores specified by the register function of *BiocParallel*. For example, the following chunk (not evaluated here), would register 4 cores, and then the two functions above, with parallel=TRUE, would split computation over these cores.

library("BiocParallel")
register(MulticoreParam(4))

We can order our results table by the smallest adjusted p value:

```
resOrdered <- res[order(res$padj),]
head(resOrdered)</pre>
```

```
## log2 fold change (MAP): condition treated vs untreated
```

```
## Wald test p-value: condition treated vs untreated
```

```
## DataFrame with 6 rows and 6 columns
```

##		baseMean	log2FoldChange	lfcSE	stat	pvalue	padj	
##		<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	
##	FBgn0039155	453	-3.71	0.160	-23.2	4.01e-119	3.09e-115	
##	FBgn0029167	2165	-2.08	0.104	-20.1	6.68e-90	2.57e-86	
##	FBgn0035085	367	-2.23	0.137	-16.3	1.89e-59	4.85e-56	
##	FBgn0029896	258	-2.21	0.159	-13.9	5.85e-44	1.13e-40	
##	FBgn0034736	118	-2.57	0.185	-13.9	8.07e-44	1.24e-40	
##	FBgn0040091	611	-1.43	0.120	-11.9	1.11e-32	1.43e-29	

We can summarize some basic tallies using the summary function.

```
summary(res)
##
## out of 11836 with nonzero total read count
## adjusted p-value < 0.1
## LFC > 0 (up) : 390, 3.3%
## LFC < 0 (down) : 408, 3.4%
## outliers [1] : 57, 0.48%
## low counts [2] : 4081, 34%
## (mean count < 9.2)
## [1] see 'cooksCutoff' argument of ?results
## [2] see 'independentFiltering' argument of ?results</pre>
```

The results function contains a number of arguments to customize the results table which is generated. Note that the results function automatically performs independent filtering based on the mean of counts for each gene, optimizing the number of genes which will have an adjusted p value below a given threshold. This will be discussed further in Section 3.8.

If a multi-factor design is used, or if the variable in the design formula has more than two levels, the contrast argument of results can be used to extract different comparisons from the *DESeqDataSet* returned by DESeq. Multi-factor designs are discussed further in Section 1.5, and the use of the contrast argument is dicussed in Section 3.2.

For advanced users, note that all the values calculated by the *DESeq2* package are stored in the *DESeqDataSet* object, and access to these values is discussed in Section 3.10.

1.4 Exploring and exporting results

1.4.1 MA-plot

In *DESeq2*, the function plotMA shows the log2 fold changes attributable to a given variable over the mean of normalized counts. Points will be colored red if the adjusted p value is less than 0.1. Points which fall out of the window are plotted as open triangles pointing either up or down.

```
plotMA(res, main="DESeq2", ylim=c(-2,2))
```

The MA-plot of log2 fold changes returned by *DESeq2* allows us to see how the shrinkage of fold changes works for genes with low counts. You can still obtain results tables which include the "unshrunken" log2 fold changes (for a simple comparison, the ratio of the mean normalized counts in the two groups). A column lfcMLE with the unshrunken maximum likelihood estimate (MLE) for the log2 fold change will be added with an additional argument to results:

```
resMLE <- results(dds, addMLE=TRUE)
head(resMLE, 4)
## log2 fold change (MAP): condition treated vs untreated</pre>
```



Figure 1: **MA-plot.** These plots show the log2 fold changes from the treatment over the mean of normalized counts, i.e. the average of counts normalized by size factors. The left plot shows the "unshrunken" log2 fold changes, while the right plot, produced by the code above, shows the shrinkage of log2 fold changes resulting from the incorporation of zero-centered normal prior. The shrinkage is greater for the log2 fold change estimates from genes with low counts and high dispersion, as can be seen by the narrowing of spread of leftmost points in the right plot.

```
## Wald test p-value: condition treated vs untreated
## DataFrame with 4 rows and 7 columns
##
                 baseMean log2FoldChange
                                              lfcMLE
                                                          lfcSE
                                                                              pvalue
                                                                      stat
##
                <numeric>
                                <numeric> <numeric> <numeric> <numeric> <numeric> <numeric>
## FBgn000003
                                   0.0346
                    0.159
                                              3.2929
                                                         0.0461
                                                                   0.7487
                                                                               0.454
## FBgn000008
                   52.226
                                              0.0281
                                                         0.2093
                                                                   0.0942
                                   0.0197
                                                                               0.925
## FBgn0000014
                    0.390
                                   0.0118
                                              0.6239
                                                        0.0622
                                                                   0.1901
                                                                               0.849
## FBgn0000015
                    0.905
                                  -0.0429
                                             -0.8135
                                                         0.1054
                                                                  -0.4076
                                                                               0.684
##
                     padj
##
                <numeric>
## FBgn000003
                       NA
## FBgn000008
                    0.988
## FBgn0000014
                       NA
## FBgn0000015
                       NA
```

After calling plotMA, one can use the function identify to interactively detect the row number of individual genes by clicking on the plot.

identify(res\$baseMean, res\$log2FoldChange)



Figure 2: **Plot of counts for one gene.** The plot of normalized counts (plus a pseudocount of $\frac{1}{2}$) either made using the plotCounts function (left) or using another plotting library (right, using *ggplot2*).

1.4.2 Plot counts

It can also be useful to examine the counts of reads for a single gene across the groups. A simple function for making this plot is plotCounts, which normalizes counts by sequencing depth and adds a pseudocount of $\frac{1}{2}$ to allow for log scale plotting. The counts are grouped by the variables in intgroup, where more than one variable can be specified. Here we specify the gene which had the smallest p value from the results table created above. You can select the gene to plot by rowname or by numeric index.

plotCounts(dds, gene=which.min(res\$padj), intgroup="condition")

For customized plotting, an argument returnData specifies that the function should only return a *data.frame* for plotting with ggplot.

1.4.3 More information on results columns

Information about which variables and tests were used can be found by calling the function mcols on the results object.

```
mcols(res)$description
```

```
## [1] "mean of normalized counts for all samples"
## [2] "log2 fold change (MAP): condition treated vs untreated"
## [3] "standard error: condition treated vs untreated"
## [4] "Wald statistic: condition treated vs untreated"
## [5] "Wald test p-value: condition treated vs untreated"
## [6] "BH adjusted p-values"
```

For a particular gene, a log2 fold change of -1 for condition treated vs untreated means that the treatment induces a change in observed expression level of $2^{-1} = 0.5$ compared to the untreated condition. If the variable of interest is continuous-valued, then the reported log2 fold change is per unit of change of that variable.

Note: some values in the results table can be set to NA, for either one of the following reasons:

- 1. If within a row, all samples have zero counts, the baseMean column will be zero, and the $\log 2$ fold change estimates, p value and adjusted p value will all be set to NA.
- 2. If a row contains a sample with an extreme count outlier then the p value and adjusted p value are set to NA. These outlier counts are detected by Cook's distance. Customization of this outlier filtering and description of functionality for replacement of outlier counts and refitting is described in Section 3.6,
- 3. If a row is filtered by automatic independent filtering, based on low mean normalized count, then only the adjusted p value is set to NA. Description and customization of independent filtering is described in Section 3.8.

The column of log2FoldChange for the default workflow will contain shrunken estimates of fold change as mentioned above. It is possible to add a column to the results table – without rerunning the analysis – which contains the unshrunken, or maximum likelihood estimates (MLE), log2 fold changes. This will add the column lfcMLE directly after log2FoldChange.

```
head(results(dds, addMLE=TRUE),4)
```

log2 fold change (MAP): condition treated vs untreated

```
## Wald test p-value: condition treated vs untreated
```

```
## DataFrame with 4 rows and 7 columns
```

##		baseMean	log2FoldChange	lfcMLE	lfcSE	stat	pvalue	
##		<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	<numeric></numeric>	
##	FBgn000003	0.159	0.0346	3.2929	0.0461	0.7487	0.454	
##	FBgn000008	52.226	0.0197	0.0281	0.2093	0.0942	0.925	
##	FBgn0000014	0.390	0.0118	0.6239	0.0622	0.1901	0.849	
##	FBgn0000015	0.905	-0.0429	-0.8135	0.1054	-0.4076	0.684	
##		padi						

##		<numeric></numeric>
##	FBgn000003	NA
##	FBgn000008	0.988
##	FBgn0000014	NA
##	FBgn0000015	NA

1.4.4 Exporting results to HTML or CSV files

An HTML report of the results with plots and sortable/filterable columns can be exported using the *ReportingTools* package on a *DESeqDataSet* that has been processed by the DESeq function. For a code example, see the "RNA-seq differential expression" vignette at the *ReportingTools* page, or the manual page for the publish method for the *DESeqDataSet* class.

A plain-text file of the results can be exported using the base *R* functions write.csv or write.delim. We suggest using a descriptive file name indicating the variable and levels which were tested.

Exporting only the results which pass an adjusted p value threshold can be accomplished with the subset function, followed by the write.csv function.

```
resSig <- subset(resOrdered, padj < 0.1)</pre>
resSig
## log2 fold change (MAP): condition treated vs untreated
## Wald test p-value: condition treated vs untreated
## DataFrame with 798 rows and 6 columns
##
                 baseMean log2FoldChange
                                               lfcSE
                                                          stat
                                                                   pvalue
                                                                                padj
##
                <numeric>
                                <numeric> <numeric> <numeric> <numeric> <numeric> <numeric>
## FBgn0039155
                      453
                                    -3.71
                                               0.160
                                                         -23.2 4.01e-119 3.09e-115
## FBgn0029167
                                    -2.08
                                               0.104
                                                         -20.1 6.68e-90 2.57e-86
                     2165
## FBgn0035085
                      367
                                    -2.23
                                               0.137
                                                         -16.3 1.89e-59 4.85e-56
## FBgn0029896
                                    -2.21
                                                         -13.9 5.85e-44 1.13e-40
                      258
                                               0.159
## FBgn0034736
                      118
                                    -2.57
                                               0.185
                                                         -13.9 8.07e-44 1.24e-40
## ...
                                      . . .
                                                            . . .
                      . . .
                                                 . . .
                                                                      . . .
                                                                                 . . .
## FBgn0000633
                     14.6
                                    0.587
                                               0.229
                                                          2.57
                                                                   0.0103
                                                                              0.0994
## FBgn0003862
                   2076.3
                                   -0.400
                                               0.156
                                                         -2.56
                                                                   0.0103
                                                                              0.0997
## FBgn0028694
                    960.2
                                    0.313
                                               0.122
                                                          2.57
                                                                   0.0103
                                                                              0.0997
## FBgn0038349
                   1275.4
                                   -0.311
                                               0.121
                                                         -2.56
                                                                   0.0103
                                                                              0.0997
## FBgn0243512
                   1135.0
                                   -0.384
                                               0.150
                                                         -2.56
                                                                   0.0103
                                                                              0.0997
```

1.5 Multi-factor designs

Experiments with more than one factor influencing the counts can be analyzed using model formula including the additional variables. The data in the *pasilla* package have a condition of interest (the column condition), as well as information on the type of sequencing which was performed (the column type), as we can see below:

```
colData(dds)
## DataFrame with 7 rows and 3 columns
##
                condition
                                 type sizeFactor
##
                 <factor>
                             <factor>
                                       <numeric>
                  treated single-read
## treated1fb
                                           1.512
## treated2fb
                  treated paired-end
                                           0.784
## treated3fb
                  treated paired-end
                                           0.896
## untreated1fb untreated single-read
                                           1.050
## untreated2fb untreated single-read
                                           1.659
## untreated3fb untreated paired-end
                                           0.712
## untreated4fb untreated paired-end
                                           0.784
```

We create a copy of the DESeqDataSet, so that we can rerun the analysis using a multi-factor design.

ddsMF <- dds

We can account for the different types of sequencing, and get a clearer picture of the differences attributable to the treatment. As condition is the variable of interest, we put it at the end of the formula. Thus the results function will by default pull the condition results unless contrast or name arguments are specified. Then we can re-run DESeq:

```
design(ddsMF) <- formula(~ type + condition)
ddsMF <- DESeq(ddsMF)</pre>
```

Again, we access the results using the results function.

```
resMF <- results(ddsMF)</pre>
head(resMF)
## log2 fold change (MAP): condition treated vs untreated
## Wald test p-value: condition treated vs untreated
## DataFrame with 6 rows and 6 columns
                 baseMean log2FoldChange
##
                                              lfcSE
                                                                   pvalue
                                                                               padj
                                                          stat
##
                <numeric>
                                <numeric> <numeric> <numeric> <numeric> <numeric> <numeric>
## FBgn000003
                    0.159
                                  0.03270
                                             0.0437
                                                        0.7478
                                                                   0.4546
                                                                                 NA
## FBgn000008
                   52.226
                                  0.01219
                                             0.2078
                                                        0.0587
                                                                   0.9532
                                                                              0.988
## FBgn0000014
                    0.390
                                             0.0563
                                                        0.1724
                                                                   0.8631
                                                                                 NA
                                  0.00971
## FBgn0000015
                                                       -0.3828
                    0.905
                                 -0.03567
                                             0.0932
                                                                   0.7019
                                                                                  NA
## FBgn0000017
                                                       -2.3331
                 2358.243
                                 -0.25675
                                             0.1100
                                                                   0.0196
                                                                              0.134
## FBgn0000018
                                 -0.06669
                                             0.1417
                                                       -0.4706
                                                                              0.884
                  221.242
                                                                   0.6380
```

It is also possible to retrieve the log2 fold changes, p values and adjusted p values of the type variable. The contrast argument of the function results takes a character vector of length three: the name of the variable, the name of the factor level for the numerator of the log2 ratio, and the name of the factor level for the denominator. The contrast argument can also take other forms, as described in the help page for results and in Section 3.2.

```
resMFType <- results(ddsMF, contrast=c("type","single-read","paired-end"))</pre>
head(resMFType)
## log2 fold change (MAP): type single-read vs paired-end
## Wald test p-value: type single-read vs paired-end
## DataFrame with 6 rows and 6 columns
##
                 baseMean log2FoldChange
                                              lfcSE
                                                                   pvalue
                                                                               padj
                                                          stat
##
                <numeric>
                                <numeric> <numeric> <numeric> <numeric> <numeric> <numeric>
## FBgn000003
                    0.159
                                 -0.02392
                                             0.0381
                                                       -0.6280
                                                                   0.5300
                                                                                  NA
## FBgn000008
                   52.226
                                             0.1974
                                                       -0.3158
                                                                   0.7521
                                                                              0.885
                                 -0.06233
## FBgn0000014
                    0.390
                                  0.00570
                                             0.0490
                                                        0.1163
                                                                   0.9074
                                                                                 NA
## FBgn0000015
                    0.905
                                 -0.05581
                                             0.0815
                                                       -0.6845
                                                                   0.4937
                                                                                  NA
## FBgn0000017
                 2358.243
                                  0.00938
                                             0.1088
                                                        0.0862
                                                                   0.9313
                                                                              0.973
## FBgn0000018
                  221.242
                                  0.26861
                                             0.1383
                                                        1.9420
                                                                   0.0521
                                                                              0.210
```

If the variable is continuous or an interaction term (see Section 3.3) then the results can be extracted using the name argument to results, where the name is one of elements returned by resultsNames(dds).

2 Data transformations and visualization

2.1 Count data transformations

In order to test for differential expression, we operate on raw counts and use discrete distributions as described in the previous Section 1.3. However for other downstream analyses – e.g. for visualization or clustering – it might be useful to work with transformed versions of the count data.

Maybe the most obvious choice of transformation is the logarithm. Since count values for a gene can be zero in some conditions (and non-zero in others), some advocate the use of *pseudocounts*, i.e. transformations of the form

$$y = \log_2(n+1)$$
 or more generally, $y = \log_2(n+n_0)$, (1)

where n represents the count values and n_0 is a positive constant.

In this section, we discuss two alternative approaches that offer more theoretical justification and a rational way of choosing the parameter equivalent to n_0 above. The *regularized logarithm* or *rlog* incorporates a prior on the sample differences [1], and the other uses the concept of variance stabilizing transformations (VST) [4, 5, 6]. Both transformations produce transformed data on the \log_2 scale which has been normalized with respect to library size.

The point of these two transformations, the *rlog* and the VST, is to remove the dependence of the variance on the mean, particularly the high variance of the logarithm of count data when the mean is low. Both *rlog* and VST use the experiment-wide trend of variance over mean, in order to transform the data to remove the experiment-wide trend. Note that we do not require or desire that all the genes have *exactly* the same variance after transformation. Indeed, in Figure 4 below, you will see that after the transformations the genes with the same mean do not have exactly the same standard deviations, but that the experiment-wide trend has flattened. It is those genes with row variance above the trend which will allow us to cluster samples into interesting groups.

2.1.1 Blind dispersion estimation

The two functions, rlog and varianceStabilizingTransformation, have an argument blind, for whether the transformation should be blind to the sample information specified by the design formula. When blind equals TRUE (the default), the functions will re-estimate the dispersions using only an intercept (design formula \sim 1). This setting should be used in order to compare samples in a manner wholly unbiased by the information about experimental groups, for example to perform sample QA (quality assurance) as demonstrated below.

However, blind dispersion estimation is not the appropriate choice if one expects that many or the majority of genes (rows) will have large differences in counts which are explanable by the experimental design, and one wishes to tranform the data for downstream analysis. In this case, using blind dispersion estimation will lead to large estimates of dispersion, as it attributes differences due to experimental design as unwanted "noise", and shrinks the tranformed values towards each other. By setting blind

to FALSE, the dispersions already estimated will be used to perform transformations, or if not present, they will be estimated using the current design formula. Note that only the fitted dispersion estimates from mean-dispersion trend line is used in the transformation. So setting blind to FALSE is still mostly unbiased by the information about the samples.

2.1.2 Extracting transformed values

The two functions return *SummarizedExperiment* objects, as the data are no longer counts. The assay function is used to extract the matrix of normalized values:

```
rld <- rlog(dds)
vsd <- varianceStabilizingTransformation(dds)
rlogMat <- assay(rld)
vstMat <- assay(vsd)</pre>
```

Note that if you have many samples, and the rlog function is taking too long, there is an argument fast=TRUE, which will perform an approximation of the rlog: instead of shrinking the samples independently, the function will find the optimal amount of shrinkage for each gene given the mean counts. The optimization is performed on the same likelihood of the data as the original rlog. The speed-up for a dataset with 100 samples is around 15x.

2.1.3 Regularized log transformation

The function rlog, stands for *regularized log*, transforming the original count data to the log2 scale by fitting a model with a term for each sample and a prior distribution on the coefficients which is estimated from the data. This is the same kind of shrinkage (sometimes referred to as regularization, or moderation) of log fold changes used by the DESeq and nbinomWaldTest, as seen in Figure 1. The resulting data contains elements defined as:

$$\log_2(q_{ij}) = \beta_{i0} + \beta_{ij}$$

where q_{ij} is a parameter proportional to the expected true concentration of fragments for gene *i* and sample *j* (see Section 4.1), β_{i0} is an intercept which does not undergo shrinkage, and β_{ij} is the sample-specific effect which is shrunk toward zero based on the dispersion-mean trend over the entire dataset. The trend typically captures high dispersions for low counts, and therefore these genes exhibit higher shrinkage from therlog.

Note that, as q_{ij} represents the part of the mean value μ_{ij} after the size factor s_j has been divided out, it is clear that the rlog transformation inherently accounts for differences in sequencing depth. Without priors, this design matrix would lead to a non-unique solution, however the addition of a prior on non-intercept betas allows for a unique solution to be found. The regularized log transformation is preferable to the variance stabilizing transformation if the size factors vary widely.



Figure 3: **VST and log2.** Graphs of the variance stabilizing transformation for sample 1, in blue, and of the transformation $f(n) = \log_2(n/s_1)$, in black. n are the counts and s_1 is the size factor for the first sample.

2.1.4 Variance stabilizing transformation

Above, we used a parametric fit for the dispersion. In this case, the closed-form expression for the variance stabilizing transformation is used by varianceStabilizingTransformation, which is derived in the file vst.pdf, that is distributed in the package alongside this vignette. If a local fit is used (option fitType="locfit" to estimateDispersions) a numerical integration is used instead.

The resulting variance stabilizing transformation is shown in Figure 3. The code that produces the figure is hidden from this vignette for the sake of brevity, but can be seen in the .Rnw or .R source file. Note that the vertical axis in such plots is the square root of the variance over all samples, so including the variance due to the experimental conditions. While a flat curve of the square root of variance over the mean may seem like the goal of such transformations, this may be unreasonable in the case of datasets with many true differences due to the experimental conditions.

2.1.5 Effects of transformations on the variance

Figure 4 plots the standard deviation of the transformed data, across samples, against the mean, using the shifted logarithm transformation (1), the regularized log transformation and the variance stabilizing transformation. The shifted logarithm has elevated standard deviation in the lower count range, and the regularized log to a lesser extent, while for the variance stabilized data the standard deviation is roughly constant along the whole dynamic range.



Figure 4: Standard deviation over mean. Per-gene standard deviation (taken across samples), against the rank of the mean, for the shifted logarithm $\log_2(n+1)$ (left), the regularized log transformation (center) and the variance stabilizing transformation (right).

```
library("vsn")
par(mfrow=c(1,3))
notAllZero <- (rowSums(counts(dds))>0)
meanSdPlot(log2(counts(dds,normalized=TRUE)[notAllZero,] + 1))
meanSdPlot(assay(rld[notAllZero,]))
meanSdPlot(assay(vsd[notAllZero,]))
```

2.2 Data quality assessment by sample clustering and visualization

Data quality assessment and quality control (i. e. the removal of insufficiently good data) are essential steps of any data analysis. These steps should typically be performed very early in the analysis of a new data set, preceding or in parallel to the differential expression testing.

We define the term *quality* as *fitness for purpose*³. Our purpose is the detection of differentially expressed genes, and we are looking in particular for samples whose experimental treatment suffered from an anormality that renders the data points obtained from these particular samples detrimental to our purpose.

2.2.1 Heatmap of the count matrix

To explore a count matrix, it is often instructive to look at it as a heatmap. Below we show how to produce such a heatmap from the raw and transformed data.

³http://en.wikipedia.org/wiki/Quality_%28business%29



Figure 5: Heatmaps showing the expression data of the 30 most highly expressed genes. The data is of raw counts (left), from regularized log transformation (center) and from variance stabilizing transformation (right).

2.2.2 Heatmap of the sample-to-sample distances

Another use of the transformed data is sample clustering. Here, we apply the dist function to the transpose of the transformed count matrix to get sample-to-sample distances. We could alternatively use the variance stabilized transformation here.



Figure 6: **Sample-to-sample distances.** Heatmap showing the Euclidean distances between the samples as calculated from the regularized log transformation.

```
distsRL <- dist(t(assay(rld)))</pre>
```

A heatmap of this distance matrix gives us an overview over similarities and dissimilarities between samples (Figure 6): We have to provide a hierarchical clustering hc to the heatmap.2 function based on the sample distances, or else the heatmap.2 function would calculate a clustering based on the distances between the rows/columns of the distance matrix.

2.2.3 Principal component plot of the samples

Related to the distance matrix of Section 2.2.2 is the PCA plot of the samples, which we obtain as follows (Figure 7).



Figure 7: **PCA plot.** PCA plot. The 7 samples shown in the 2D plane spanned by their first two principal components. This type of plot is useful for visualizing the overall effect of experimental covariates and batch effects.

```
plotPCA(rld, intgroup=c("condition", "type"))
```

It is also possible to customize the PCA plot using the ggplot function.

```
data <- plotPCA(rld, intgroup=c("condition", "type"), returnData=TRUE)
percentVar <- round(100 * attr(data, "percentVar"))
ggplot(data, aes(PC1, PC2, color=condition, shape=type)) +
   geom_point(size=3) +
   xlab(paste0("PC1: ",percentVar[1],"% variance")) +
   ylab(paste0("PC2: ",percentVar[2],"% variance"))</pre>
```



Figure 8: PCA plot. PCA plot customized using the ggplot2 library.

3 Variations to the standard workflow

3.1 Wald test individual steps

The function DESeq runs the following functions in order:

```
dds <- estimateSizeFactors(dds)
dds <- estimateDispersions(dds)
dds <- nbinomWaldTest(dds)</pre>
```

3.2 Contrasts

A contrast is a linear combination of estimated log2 fold changes, which can be used to test if differences between groups are equal to zero. The simplest use case for contrasts is an experimental design containing a factor with three levels, say A, B and C. Contrasts enable the user to generate results for all 3 possible differences: log2 fold change of B vs A, of C vs A, and of C vs B (the other three possible pairs will simply have $-1\times$ the log2 fold changes of these three).

In order to fit models with "shrunken" log2 fold changes in a manner which is independent to the choice of base level, *DESeq2* uses "expanded model matrices", described further in Section 4.5. The expanded model matrices include a coefficient for each level of the factors in addition to an intercept. The contrast argument of results function is again used to extract test results of log2 fold changes of interest.

Log2 fold changes can also be added and subtracted by providing a list to the contrast argument with two elements: the names of the log2 fold changes to add, and the names of the log2 fold changes to subtract. Alternatively, a numeric vector of the length of resultsNames(dds) can be provided, for

manually specifying the linear combination of terms. Demonstrations of the use of contrasts for various designs can be found in the examples section of the help page for the results function. The formula that is used to generate the contrasts can be found in Section 4.4.

3.3 Interactions

Interaction terms can be added to the design formula, in order to test, for example, if the log2 fold change attributable to a given condition is different based on a second variable, for example if the treatment effect of a drug is differs based on another grouping variable like sex or species. Interactions are specified in R formula using a colon, :, between the two variables names. Demonstrations of extracting results from an interaction model are shown in the examples section of the help page for results.

3.4 Time-series experiments

There are a number of ways to analyze time-series experiments, depending on the biological question of interest. In order to test for any differences over multiple time points, once can use a design including the time factor, and then test using the likelihood ratio test as described in Section 3.5, where the time factor is removed in the reduced formula. For a control and treatment time series, one can use a design formula containing the condition factor, the time factor, and the interaction of the two. In this case, using the likelihood ratio test with a reduced model which does not contain the interaction terms will test whether the condition induces a change in gene expression at any time point after the base-level time point (time 0). An example of the later analysis is provided in an RNA-Seq workflow on the Bioconductor website: http://www.bioconductor.org/help/workflows/rnaseqGene/.

3.5 Likelihood ratio test

DESeq2 offers two kinds of hypothesis tests: the Wald test, where we use the estimated standard error of a log2 fold change to test if it is equal to zero, and the likelihood ratio test (LRT). The LRT examines two models for the counts, a *full* model with a certain number of terms and a *reduced* model, in which some of the terms of the *full* model are removed. The test determines if the increased likelihood of the data using the extra terms in the *full* model is more than expected if those extra terms are truly zero.

The LRT is therefore useful for testing multiple terms at once, for example testing 3 or more levels of a factor at once, or all interactions between two variables. The LRT for count data is conceptually similar to an analysis of variance (ANOVA) calculation in linear regression, except that in the case of the Negative Binomial GLM, we use an analysis of deviance (ANODEV), where the *deviance* captures the difference in likelihood between a full and a reduced model.

The likelihood ratio test can be specified using the test argument to DESeq, which substitutes nbinomWaldTest with nbinomLRT. In this case, the user needs to provide a reduced formula, e.g. one in which a number of terms from design(dds) are removed. The degrees of freedom for the test is obtained from the difference between the number of parameters in the two models.

3.6 Approach to count outliers

RNA-Seq data sometimes contain isolated instances of very large counts that are apparently unrelated to the experimental or study design, and which may be considered outliers. There are many reasons why outliers can arise, including rare technical or experimental artifacts, read mapping problems in the case of genetically differing samples, and genuine, but rare biological events. In many cases, users appear primarily interested in genes that show a consistent behavior, and this is the reason why by default, genes that are affected by such outliers are set aside by *DESeq2*, or if there are sufficient samples, outlier counts are replaced for model fitting. These two behaviors are described below.

The DESeq function (and nbinomWaldTest/nbinomLRT functions) calculates, for every gene and for every sample, a diagnostic test for outliers called *Cook's distance*. Cook's distance is a measure of how much a single sample is influencing the fitted coefficients for a gene, and a large value of Cook's distance is intended to indicate an outlier count. The Cook's distances are stored as a matrix available in assays(dds)[["cooks"]].

The results function automatically flags genes which contain a Cook's distance above a cutoff for samples which have 3 or more replicates. The p values and adjusted p values for these genes are set to NA. At least 3 replicates are required for flagging, as it is difficult to judge which sample might be an outlier with only 2 replicates.

With many degrees of freedom – i. e., many more samples than number of parameters to be estimated – it is undesirable to remove entire genes from the analysis just because their data include a single count outlier. When there are 7 or more replicates for a given sample, the DESeq function will automatically replace counts with large Cook's distance with the trimmed mean over all samples, scaled up by the size factor or normalization factor for that sample. This approach is conservative, it will not lead to false positives, as it replaces the outlier value with the value predicted by the null hypothesis.

The default Cook's distance cutoff for the two behaviors described above depends on the sample size and number of parameters to be estimated. The default is to use the 99% quantile of the F(p, m-p) distribution (with p the number of parameters including the intercept and m number of samples). The default for gene flagging can be modified using the cooksCutoff argument to the results function. The gene flagging functionality can be disabled by setting cooksCutoff to FALSE or Inf. The automatic outlier replacement performed by DESeq can be disabled by setting the minReplicatesForReplace argument to Inf.

DESeq automatically replaces outliers if there are sufficient replicates and a row contains a count with very high Cook's distance. DESeq preserves the original counts in counts(dds) saving the replacement counts as a matrix named replaceCounts in assays(dds).

3.7 Dispersion plot and fitting alternatives

Plotting the dispersion estimates is a useful diagnostic. The dispersion plot in Figure 9 is typical, with the final estimates shrunk from the gene-wise estimates towards the fitted estimates. Some gene-wise estimates are flagged as outliers and not shrunk towards the fitted value, (this outlier detection is described in the man page for estimateDispersionsMAP). The amount of shrinkage can be more or



Figure 9: **Dispersion plot.** The dispersion estimate plot shows the gene-wise estimates (black), the fitted values (red), and the final maximum *a posteriori* estimates used in testing (blue).

less than seen here, depending on the sample size, the number of coefficients, the row mean and the variability of the gene-wise estimates.

plotDispEsts(dds)

3.7.1 Local or mean dispersion fit

A local smoothed dispersion fit is automatically substituted in the case that the parametric curve doesn't fit the observed dispersion mean relationship. This can be prespecified by providing the argument fitType="local" to either DESeq or estimateDispersions. Additionally, using the mean of genewise disperion estimates as the fitted value can be specified by providing the argument fitType="mean".

3.7.2 Supply a custom dispersion fit

Any fitted values can be provided during dispersion estimation, using the lower-level functions described in the manual page for estimateDispersionsGeneEst. In the code chunk below, we store the genewise estimates which were already calculated and saved in the metadata column dispGeneEst. Then we calculate the median value of the dispersion estimates above a threshold, and save these values as the fitted dispersions, using the replacement function for dispersionFunction. In the last line, the function estimateDispersionsMAP, uses the fitted dispersions to generate maximum *a posteriori* (MAP) estimates of dispersion.

```
ddsCustom <- dds
useForMedian <- mcols(ddsCustom)$dispGeneEst > 1e-7
medianDisp <- median(mcols(ddsCustom)$dispGeneEst[useForMedian],na.rm=TRUE)
dispersionFunction(ddsCustom) <- function(mu) medianDisp
ddsCustom <- estimateDispersionsMAP(ddsCustom)</pre>
```

3.8 Independent filtering of results

The results function of the *DESeq2* package performs independent filtering by default using the mean of normalized counts as a filter statistic. A threshold on the filter statistic is found which optimizes the number of adjusted p values lower than a significance level alpha (we use the standard variable name for significance level, though it is unrelated to the dispersion parameter α). The theory behind independent filtering is discussed in greater detail in Section 4.6. The adjusted p values for the genes which do not pass the filter threshold are set to NA.

The independent filtering is performed using the filtered_p function of the *genefilter* package, and all of the arguments of filtered_p can be passed to the results function. The filter threshold value and the number of rejections at each quantile of the filter statistic are available as attributes of the object returned by results. For example, we can visualize the optimization by plotting the filterNumRej attribute of the results object, as seen in Figure 10. Note that if the maximum number of rejections is very small such that the line of rejections over filter threshold appears noisy, the expected false discovery rate might not be held exactly for this small set.

```
attr(res,"filterThreshold")
## 46.5%
## 9.2
plot(attr(res,"filterNumRej"),type="b",
    ylab="number of rejections")
```

Independent filtering can be turned off by setting independentFiltering to FALSE.

```
resNoFilt <- results(dds, independentFiltering=FALSE)
addmargins(table(filtering=(res$padj < .1), noFiltering=(resNoFilt$padj < .1)))
## noFiltering
## filtering FALSE TRUE Sum
## FALSE 6900 0 6900
## TRUE 108 690 798
## Sum 7008 690 7698</pre>
```



Figure 10: **Independent filtering.** The results function maximizes the number of rejections (adjusted *p* value less than a significance level), over theta, the quantiles of a filtering statistic (in this case, the mean of normalized counts).

3.9 Tests of log2 fold change above or below a threshold

It is also possible to provide thresholds for constructing Wald tests of significance. Two arguments to the results function allow for threshold-based Wald tests: lfcThreshold, which takes a numeric of a non-negative threshold value, and altHypothesis, which specifies the kind of test. Note that the *alternative hypothesis* is specified by the user, i.e. those genes which the user is interested in finding, and the test provides p values for the null hypothesis, the complement of the set defined by the alternative. The altHypothesis argument can take one of the following four values, where β is the log2 fold change specified by the name argument:

- greaterAbs $|\beta| > lfcThreshold$ tests are two-tailed
- lessAbs $|\beta| < \text{lfcThreshold}$ p values are the maximum of the upper and lower tests
- greater $\beta > lfcThreshold$
- less $\beta < -lfcThreshold$

The test altHypothesis="lessAbs" requires that the user have run DESeq with the argument betaPrior=FALSE. To understand the reason for this requirement, consider that during hypothesis testing, the null hypothesis is favored unless the data provide strong evidence to reject the null. For this test, including a zero-centered prior on log fold change would favor the alternative hypothesis, shrinking log fold changes toward zero. Removing the prior on log fold changes for tests of small log fold change allows for detection of only those genes where the data alone provides evidence against the null.

The four possible values of altHypothesis are demonstrated in the following code and visually by MA-plots in Figure 11. First we run DESeq and specify betaPrior=FALSE in order to demonstrate altHypothesis="lessAbs".

ddsNoPrior <- DESeq(dds, betaPrior=FALSE)</pre>

In order to produce results tables for the following tests, the same arguments (except ylim) would be provided to the results function.

```
par(mfrow=c(2,2),mar=c(2,2,1,1))
y1 <- c(-2.5,2.5)

resGA <- results(dds, lfcThreshold=.5, altHypothesis="greaterAbs")
resL <- results(ddsNoPrior, lfcThreshold=.5, altHypothesis="lessAbs")
resL <- results(dds, lfcThreshold=.5, altHypothesis="greater")
resL <- results(dds, lfcThreshold=.5, altHypothesis="less")

plotMA(resGA, ylim=yl)
abline(h=c(-.5,.5),col="dodgerblue",lwd=2)
plotMA(resG, ylim=yl)
abline(h=.5,col="dodgerblue",lwd=2)
plotMA(resL, ylim=yl)
abline(h=.5,col="dodgerblue",lwd=2)
plotMA(resL, ylim=yl)
abline(h=-.5,col="dodgerblue",lwd=2)</pre>
```

3.10 Access to all calculated values

All row-wise calculated values (intermediate dispersion calculations, coefficients, standard errors, etc.) are stored in the *DESeqDataSet* object, e.g. dds in this vignette. These values are accessible by calling mcols on dds. Descriptions of the columns are accessible by two calls to mcols.

```
mcols(dds,use.names=TRUE)[1:4,1:4]
```

DataFrame with 4 rows and 4 columns ## gene baseMean baseVar allZero ## <factor> <numeric> <numeric> <logical> ## FBgn0000003 FBgn0000003 0.159 0.178 FALSE ## FBgn0000008 FBgn0000008 52.226 154.611 FALSE ## FBgn0000014 FBgn0000014 0.390 0.444 FALSE ## FBgn0000015 FBgn0000015 0.905 0.799 FALSE

here using substr() only for display purposes
substr(names(mcols(dds)),1,10)

##	[1]	"gene"	"baseMean"	"baseVar"	"allZero"	"dispGeneEs"
##	[6]	"dispFit"	"dispersion"	"dispIter"	"dispOutlie"	"dispMAP"



Figure 11: **MA-plots of tests of log2 fold change with respect to a threshold value.** Going left to right across rows, the tests are for altHypothesis = "greaterAbs", "lessAbs", "greater", and "less".

```
## [11] "Intercept" "conditionu" "conditiont" "SE_Interce" "SE_conditi"
## [16] "SE_conditi" "MLE_Interc" "MLE_condit" "WaldStatis" "WaldStatis"
## [21] "WaldStatis" "WaldPvalue" "WaldPvalue" "WaldPvalue" "betaConv"
                     "deviance"
## [26] "betaIter"
                                  "maxCooks"
mcols(mcols(dds), use.names=TRUE)[1:4,]
## DataFrame with 4 rows and 2 columns
                                                            description
##
                    type
##
                                                            <character>
             <character>
## gene
                   input
## baseMean intermediate
                             mean of normalized counts for all samples
## baseVar intermediate variance of normalized counts for all samples
## allZero intermediate
                                        all counts for a gene are zero
```

The mean values $\mu_{ij} = s_j q_{ij}$ and the Cook's distances for each gene and sample are stored as matrices in the assays slot:

head(assays(dds)[["mu"]])

##	treated1fb	treated2fb	treated3fb	untreated1fb	untreated2fb
## FBgn000003	0.232	0.121	0.138	0.158	0.249

##	FBgn000008	79.292	41.141	46.989	54.328	85.815
##	FBgn0000014	0.593	0.308	0.352	0.409	0.646
##	FBgn0000015	1.287	0.668	0.762	0.921	1.454
##	FBgn0000017	3208.117	1664.554	1901.137	2659.781	4201.345
##	FBgn0000018	322.080	167.113	190.865	240.407	379.743
##		untreated3fb	untreated	4fb		
##	FBgn000003	0.107	0.	118		
##	FBgn000008	36.828	40.	552		
##	FBgn0000014	0.277	0.	305		
##	FBgn0000015	0.624	0.	687		
##	FBgn0000017	1803.025	1985.	333		
##	FBgn0000018	162.968	179.	446		

head(assays(dds)[["cooks"]])

##		treated1fb	treated2fb	treated3fb	untreated1fb	untreated2fb
##	FBgn000003	0.101997	0.040425	1.93004	0.0350	0.055309
##	FBgn000008	0.001619	0.052975	0.03061	0.0522	0.005116
##	FBgn0000014	1.496984	0.101639	0.12473	0.0947	0.186488
##	FBgn0000015	0.026910	0.168946	0.02097	0.1773	0.031421
##	FBgn0000017	0.000591	0.000259	0.00645	0.0516	0.078397
##	FBgn0000018	0.366428	0.130161	0.08158	0.3468	0.000767
##		untreated3f	b untreated	d4fb		
##	FBgn000003	0.023	7 0.02	2614		
##	FBgn000008	0.413	5 0.25	5905		
##	FBgn0000014	0.346	3 0.05	5955		
##	FBgn0000015	0.036	2 0.42	2199		
##	FBgn0000017	0.160	0 0.14	4812		
##	FBgn0000018	0.214	8 0.00	0708		

The dispersions α_i can be accessed with the dispersions function.

```
head(dispersions(dds))
```

[1] 10.0000 0.0538 6.3982 1.7320 0.0133 0.0220

which is the same as
head(mcols(dds)\$dispersion)

[1] 10.0000 0.0538 6.3982 1.7320 0.0133 0.0220

The size factors s_j are accessible via sizeFactors:

```
sizeFactors(dds)
```

##	treated1fb	treated2fb	treated3fb	untreated1fb	${\tt untreated2fb}$	untreated3fb
##	1.512	0.784	0.896	1.050	1.659	0.712
##	untreated4fb					
##	0.784					

For advanced users, we also include a convenience function coef for extracting the matrix of coefficients $[\beta_{ir}]$ for all genes *i* and parameters *r*, as in the formula in Section 4.1. This function can also return a matrix of standard errors, see ?coef. The columns of this matrix correspond to the effects returned by resultsNames. Note that the results function is best for building results tables with *p* values and adjusted *p* values.

```
head(coef(dds))
```

##		Intercept	conditionuntreated	conditiontreated
##	FBgn000003	-2.719	-0.01728	0.01727
##	FBgn000008	5.703	-0.00986	0.00986
##	FBgn0000014	-1.355	-0.00591	0.00591
##	FBgn0000015	-0.211	0.02147	-0.02147
##	FBgn0000017	11.179	0.12768	-0.12768
##	FBgn0000018	7.787	0.05192	-0.05192

The beta prior variance σ_r^2 is stored as an attribute of the *DESeqDataSet*:

```
attr(dds, "betaPriorVar")
## Intercept conditionuntreated conditiontreated
## 1.00e+06 1.05e-01 1.05e-01
```

The dispersion prior variance σ_d^2 is stored as an attribute of the dispersion function:

```
dispersionFunction(dds)
```

```
## function (q)
## coefs[1] + coefs[2]/q
## <environment: 0xe7d5a88>
## attr(,"coefficients")
## asymptDisp extraPois
                  2.5652
       0.0154
##
## attr(,"fitType")
## [1] "parametric"
## attr(,"varLogDispEsts")
## [1] 0.961
## attr(,"dispPriorVar")
## [1] 0.47
attr(dispersionFunction(dds), "dispPriorVar")
## [1] 0.47
```

3.11 Sample-/gene-dependent normalization factors

In some experiments, there might be gene-dependent dependencies which vary across samples. For instance, GC-content bias or length bias might vary across samples coming from different labs or

processed at different times. We use the terms "normalization factors" for a gene \times sample matrix, and "size factors" for a single number per sample. Incorporating normalization factors, the mean parameter μ_{ij} from Section 4.1 becomes:

$$\mu_{ij} = NF_{ij}q_{ij}$$

with normalization factor matrix NF having the same dimensions as the counts matrix K. This matrix can be incorporated as shown below. We recommend providing a matrix with row-wise geometric means of 1, so that the mean of normalized counts for a gene is close to the mean of the unnormalized counts. This can be accomplished by dividing out the current row geometric means.

```
normFactors <- normFactors / exp(rowMeans(log(normFactors)))
normalizationFactors(dds) <- normFactors</pre>
```

These steps then replace estimateSizeFactors in the steps described in Section 3.1. Normalization factors, if present, will always be used in the place of size factors.

The methods provided by the *cqn* or *EDASeq* packages can help correct for GC or length biases. They both describe in their vignettes how to create matrices which can be used by *DESeq2*. From the formula above, we see that normalization factors should be on the scale of the counts, like size factors, and unlike offsets which are typically on the scale of the predictors (i.e. the logarithmic scale for the negative binomial GLM). At the time of writing, the transformation from the matrices provided by these packages should be:

cqnOffset <- cqnObject\$glm.offset
cqnNormFactors <- exp(cqnOffset)
EDASeqNormFactors <- exp(-1 * EDASeqOffset)</pre>

4 Theory behind DESeq2

4.1 The DESeq2 model

The *DESeq2* model and all the steps taken in the software are described in detail in our pre-print [1], and we include the formula and descriptions in this section as well. The differential expression analysis in *DESeq2* uses a generalized linear model of the form:

$$K_{ij} \sim \text{NB}(\mu_{ij}, \alpha_i)$$

 $\mu_{ij} = s_j q_{ij}$
 $\log_2(q_{ij}) = x_j \beta_i$

where counts K_{ij} for gene *i*, sample *j* are modeled using a negative binomial distribution with fitted mean μ_{ij} and a gene-specific dispersion parameter α_i . The fitted mean is composed of a sample-specific size factor s_j^4 and a parameter q_{ij} proportional to the expected true concentration of fragments for sample *j*. The coefficients β_i give the log2 fold changes for gene *i* for each column of the model matrix *X*.

By default these log2 fold changes are the maximum *a priori* estimates after incorporating a zerocentered Normal prior – in the software referred to as a β -prior – hence *DESeq2* provides "moderated" log2 fold change estimates. Dispersions are estimated using expected mean values from the maximum likelihood estimate of log2 fold changes, and optimizing the Cox-Reid adjusted profile likelihood, as first implemented for RNA-Seq data in *edgeR* [7, 8]. The steps performed by the DESeq function are documented in its manual page; briefly, they are:

- 1. estimation of size factors s_i by estimateSizeFactors
- 2. estimation of dispersion α_i by estimateDispersions
- 3. negative binomial GLM fitting for β_i and Wald statistics by nbinomWaldTest

For access to all the values calculated during these steps, see Section 3.10

4.2 Changes compared to the DESeq package

The main changes in the package *DESeq2*, compared to the (older) version *DESeq*, are as follows:

- *SummarizedExperiment* is used as the superclass for storage of input data, intermediate calculations and results.
- Maximum a posteriori estimation of GLM coefficients incorporating a zero-centered Normal prior with variance estimated from data (equivalent to Tikhonov/ridge regularization). This adjustment has little effect on genes with high counts, yet it helps to moderate the otherwise large variance in log2 fold change estimates for genes with low counts or highly variable counts.

⁴The model can be generalized to use sample- **and** gene-dependent normalization factors, see Appendix 3.11.

- Maximum *a posteriori* estimation of dispersion replaces the sharingMode options fit-only or maximum of the previous version of the package. This is similar to the dispersion estimation methods of DSS [9].
- All estimation and inference is based on the generalized linear model, which includes the two condition case (previously the *exact test* was used).
- The Wald test for significance of GLM coefficients is provided as the default inference method, with the likelihood ratio test of the previous version still available.
- It is possible to provide a matrix of sample-/gene-dependent normalization factors (Section 3.11).
- Automatic independent filtering on the mean of normalized counts (Section 4.6).
- Automatic outlier detection and handling (Section 4.3).

4.3 Count outlier detection

DESeq2 relies on the negative binomial distribution to make estimates and perform statistical inference on differences. While the negative binomial is versatile in having a mean and dispersion parameter, extreme counts in individual samples might not fit well to the negative binomial. For this reason, we perform automatic detection of count outliers. We use Cook's distance, which is a measure of how much the fitted coefficients would change if an individual sample were removed [10]. For more on the implementation of Cook's distance see Section 3.6 and the manual page for the results function. Below we plot the maximum value of Cook's distance for each row over the rank of the test statistic to justify its use as a filtering criterion.

```
W <- res$stat
maxCooks <- apply(assays(dds)[["cooks"]],1,max)
idx <- !is.na(W)
plot(rank(W[idx]), maxCooks[idx], xlab="rank of Wald statistic",
     ylab="maximum Cook's distance per gene",
     ylim=c(0,5), cex=.4, col=rgb(0,0,0,.3))
m <- ncol(dds)
p <- 3
abline(h=qf(.99, p, m - p))</pre>
```

4.4 Contrasts

Contrasts can be calculated for a *DESeqDataSet* object for which the GLM coefficients have already been fit using the Wald test steps (DESeq with test="Wald" or using nbinomWaldTest). The vector of coefficients β is left multiplied by the contrast vector c to form the numerator of the test statistic. The denominator is formed by multiplying the covariance matrix Σ for the coefficients on either side by the contrast vector c. The square root of this product is an estimate of the standard error for the contrast. The contrast statistic is then compared to a normal distribution as are the Wald statistics for the *DESeq2* package.



Figure 12: **Cook's distance.** Plot of the maximum Cook's distance per gene over the rank of the Wald statistics for the condition. The two regions with small Cook's distances are genes with a single count in one sample. The horizontal line is the default cutoff used for 7 samples and 3 estimated parameters.

$$W = \frac{c^t \beta}{\sqrt{c^t \Sigma c}}$$

4.5 Expanded model matrices

As mentioned in Section 3.2, *DESeq2* uses "expanded model matrices" with the log2 fold change prior, in order to produce log2 fold change estimates and test results which are independent of the choice of base level. These model matrices differ from the standard model matrices, in that they have an indicator column (and therefore a coefficient) for each level of factors in the design formula in addition to an intercept. Expanded model matrices are not used without the log2 fold change prior or in the case of designs with 2 level factors and an interaction term.

These matrices are therefore not full rank, but a coefficient vector β_i can still be found due to the zerocentered prior on non-intercept coefficients. The prior variance for the log2 fold changes is calculated by first generating maximum likelihood estimates for a standard model matrix. The prior variance for each level of a factor is then set as the average of the mean squared maximum likelihood estimates for each level and every possible contrast, such that that this prior value will be base level independent. The contrast argument of the results function is again used in order to generate comparisons of interest.

4.6 Independent filtering and multiple testing

4.6.1 Filtering criteria

The goal of independent filtering is to filter out those tests from the procedure that have no, or little chance of showing significant evidence, without even looking at their test statistic. Typically, this results in increased detection power at the same experiment-wide type I error. Here, we measure experiment-wide type I error in terms of the false discovery rate.

A good choice for a filtering criterion is one that

- 1. is statistically independent from the test statistic under the null hypothesis,
- 2. is correlated with the test statistic under the alternative, and
- 3. does not notably change the dependence structure –if there is any– between the tests that pass the filter, compared to the dependence structure between the tests before filtering.

The benefit from filtering relies on property 2, and we will explore it further in Section 4.6.2. Its statistical validity relies on property 1 - which is simple to formally prove for many combinations of filter criteria with test statistics- and 3, which is less easy to theoretically imply from first principles, but rarely a problem in practice. We refer to [11] for further discussion of this topic.

A simple filtering criterion readily available in the results object is the mean of normalized counts irrespective of biological condition (Figure 13). Genes with very low counts are not likely to see significant differences typically due to high dispersion. For example, we can plot the $-\log_{10} p$ values from all genes over the normalized mean counts.

```
plot(res$baseMean+1, -log10(res$pvalue),
    log="x", xlab="mean of normalized counts",
    ylab=expression(-log[10](pvalue)),
    ylim=c(0,30),
    cex=.4, col=rgb(0,0,0,.3))
```

4.6.2 Why does it work?

Consider the p value histogram in Figure 14. It shows how the filtering ameliorates the multiple testing problem – and thus the severity of a multiple testing adjustment – by removing a background set of hypotheses whose p values are distributed more or less uniformly in [0, 1].

```
use <- res$baseMean > attr(res,"filterThreshold")
table(use)
## use
## FALSE TRUE
## 6728 7742
```



mean of normalized counts

Figure 13: Mean counts as a filter statistic. The mean of normalized counts provides an independent statistic for filtering the tests. It is independent because the information about the variables in the design formula is not used. By filtering out genes which fall on the left side of the plot, the majority of the low p values are kept.

4.6.3 Diagnostic plots for multiple testing

The Benjamini-Hochberg multiple testing adjustment procedure [12] has a simple graphical illustration, which we produce in the following code chunk. Its result is shown in the left panel of Figure 15.

```
resFilt <- res[use & !is.na(res$pvalue),]
orderInPlot <- order(resFilt$pvalue)
showInPlot <- (resFilt$pvalue[orderInPlot] <= 0.08)</pre>
```



Figure 14: **Histogram of p values for all tests.** The area shaded in blue indicates the subset of those that pass the filtering, the area in khaki those that do not pass.

Schweder and Spjøtvoll [13] suggested a diagnostic plot of the observed *p*-values which permits estimation of the fraction of true null hypotheses. For a series of hypothesis tests H_1, \ldots, H_m with *p*-values p_i , they suggested plotting

$$(1 - p_i, N(p_i))$$
 for $i \in 1, ..., m$, (2)

where N(p) is the number of *p*-values greater than *p*. An application of this diagnostic plot to resFilt\$pvalue is shown in the right panel of Figure 15. When all null hypotheses are true, the *p*-values are each uniformly distributed in [0,1], Consequently, the cumulative distribution function of (p_1,\ldots,p_m) is expected to be close to the line F(t) = t. By symmetry, the same applies to $(1-p_1,\ldots,1-p_m)$. When (without loss of generality) the first m_0 null hypotheses are true and the other $m-m_0$ are false, the cumulative distribution function of $(1-p_1,\ldots,1-p_m)$ is again expected to be close to a function $F_1(t)$ which stays below F_0 but shows a steep increase towards 1 as t approaches 1. In practice, we do not know which of the null hypotheses are

true, so we can only observe a mixture whose cumulative distribution function is expected to be close to

$$F(t) = \frac{m_0}{m} F_0(t) + \frac{m - m_0}{m} F_1(t).$$
(3)

Such a situation is shown in the right panel of Figure 15. If $F_1(t)/F_0(t)$ is small for small t, then the mixture fraction $\frac{m_0}{m}$ can be estimated by fitting a line to the left-hand portion of the plot, and then noting its height on the right. Such a fit is shown by the red line in the right panel of Figure 15.

```
plot(1-resFilt$pvalue[orderInPlot],
                (length(resFilt$pvalue)-1):0, pch=".",
                     xlab=expression(1-p[i]), ylab=expression(N(p[i])))
abline(a=0, slope, col="red3", lwd=2)
```



Figure 15: Left: illustration of the Benjamini-Hochberg multiple testing adjustment procedure [12]. The black line shows the *p*-values (*y*-axis) versus their rank (*x*-axis), starting with the smallest *p*-value from the left, then the second smallest, and so on. Only the first 1587 *p*-values are shown. The red line is a straight line with slope α/n , where n = 7698 is the number of tests, and $\alpha = 0.1$ is a target false discovery rate (FDR). FDR is controlled at the value α if the genes are selected that lie to the left of the rightmost intersection between the red and black lines: here, this results in 798 genes. *Right:* Schweder and Spjøtvoll plot, as described in the text. For both of these plots, the *p*-values resFilt\$pvalues from Section 4.6.1 were used as a starting point. Analogously, one can produce these types of plots for any set of *p*-values, for instance those from the previous sections.

5 Frequently asked questions

5.1 How can I get support for DESeq2?

We welcome questions about our software, and want to ensure that we eliminate issues if and when they appear. We have a few requests to optimize the process:

- all questions should take place on the Bioconductor support site: https://support.bioconductor. org, which serves as a repository of questions and answers. This helps to save the developers' time in responding to similar questions. Make sure to tag your post with "deseq2". It is often very helpful in addition to describe the aim of your experiment.
- before posting, first search the Bioconductor support site mentioned above for past threads which might have answered your question.
- if you have a question about the behavior of a function, read the sections of the manual page for this function by typing a question mark and the function name, e.g. ?results. We spend a lot of time documenting individual functions and the exact steps that the software is performing.
- include all of your R code, especially the creation of the DESeqDataSet and the design formula. Include complete warning or error messages, and conclude your message with the full output of sessionInfo().
- if possible, include the output of as.data.frame(colData(dds)), so that we can have a sense of the experimental setup. If this contains confidential information, you can replace the levels of those factors using levels().

5.2 Why are some *p* values set to NA?

See the details in Section 1.4.3.

5.3 How can I get unfiltered DESeq results?

Users can obtain unfiltered GLM results, i.e. without outlier removal or independent filtering with the following call:

```
dds <- DESeq(dds, minReplicatesForReplace=Inf)
res <- results(dds, cooksCutoff=FALSE, independentFiltering=FALSE)</pre>
```

In this case, the only p values set to NA are those from genes with all counts equal to zero.

5.4 How do I use the variance stabilized or rlog transformed data for differential testing?

The variance stabilizing and rlog transformations are provided for applications other than differential testing, for example clustering of samples or other machine learning applications. For differential testing

we recommend the DESeq function applied to raw counts as outlined in Section 1.3.

5.5 Can I use DESeq2 to analyze paired samples?

Yes, you should use a multi-factor design which includes the sample information as a term in the design formula. This will account for differences between the samples while estimating the effect due to the condition. The condition of interest should go at the end of the design formula. See Section 1.5.

5.6 Can I run DESeq2 to contrast the levels of 100 groups?

DESeq2 will work with any kind of design specified using the R formula. We enourage users to consider exploratory data analysis such as principal components analysis as described in Section 2.2.3, rather than performing statistical testing of all combinations of dozens of groups.

As a speed concern with fitting very large models, note that each additional level of a factor in the design formula adds another parameter to the GLM which is fit by *DESeq2*. Users might consider first removing genes with very few reads, e.g. genes with row sum of 1, as this will speed up the fitting procedure.

5.7 Can I use DESeq2 to analyze a dataset without replicates?

If a *DESeqDataSet* is provided with an experimental design without replicates, a message is printed, that the samples are treated as replicates for estimation of dispersion. More details can be found in the manual page for ?DESeq.

5.8 How can I include a continuous covariate in the design formula?

Continuous covariates can be included in the design formula in the same manner as factorial covariates. Continuous covariates might make sense in certain experiments, where a constant fold change might be expected for each unit of the covariate. However, in many cases, more meaningful results can be obtained by cutting continuous covariates into a factor defined over a small number of bins (e.g. 3-5). In this way, the average effect of each group is controlled for, regardless of the trend over the continuous covariates. In R, *numeric* vectors can be converted into *factors* using the function cut.

5.9 What are the exact steps performed by DESeq()?

See the manual page for DESeq, which links to the subfunctions which are called in order, where complete details are listed.

6 Acknowledgments

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7 Session Info

- R version 3.1.2 (2014-10-31), x86_64-unknown-linux-gnu
- Locale: LC_CTYPE=en_US.UTF-8, LC_NUMERIC=C, LC_TIME=en_US.UTF-8, LC_COLLATE=C, LC_MONETARY=en_US.UTF-8, LC_MESSAGES=en_US.UTF-8, LC_PAPER=en_US.UTF-8, LC_NAME=C, LC_ADDRESS=C, LC_TELEPHONE=C, LC_MEASUREMENT=en_US.UTF-8, LC_IDENTIFICATION=C
- Base packages: base, datasets, grDevices, graphics, methods, parallel, stats, stats4, utils
- Other packages: Biobase 2.26.0, BiocGenerics 0.12.1, DESeq2 1.6.3, GenomeInfoDb 1.2.3, GenomicRanges 1.18.3, IRanges 2.0.1, RColorBrewer 1.1-2, Rcpp 0.11.3, RcppArmadillo 0.4.550.1.0, S4Vectors 0.4.0, airway 1.0.0, ggplot2 1.0.0, gplots 2.15.0, knitr 1.8, pasilla 0.5.1, vsn 3.34.0
- Loaded via a namespace (and not attached): AnnotationDbi 1.28.1, BBmisc 1.8, BatchJobs 1.5, BiocInstaller 1.16.1, BiocParallel 1.0.0, BiocStyle 1.4.1, DBI 0.3.1, DESeq 1.18.0, Formula 1.1-2, Hmisc 3.14-6, KernSmooth 2.23-13, MASS 7.3-35, RSQLite 1.0.0, XML 3.98-1.1, XVector 0.6.0, acepack 1.3-3.3, affy 1.44.0, affyio 1.34.0, annotate 1.44.0, base64enc 0.1-2, bitops 1.0-6, brew 1.0-6, caTools 1.17.1, checkmate 1.5.1, cluster 1.15.3, codetools 0.2-9, colorspace 1.2-4, digest 0.6.6, evaluate 0.5.5, fail 1.2, foreach 1.4.2, foreign 0.8-61, formatR 1.0, gdata 2.13.3, genefilter 1.48.1, geneplotter 1.44.0, grid 3.1.2, gtable 0.1.2, gtools 3.4.1, highr 0.4, iterators 1.0.7, labeling 0.3, lattice 0.20-29, latticeExtra 0.6-26, limma 3.22.1, locfit 1.5-9.1, munsell 0.4.2, nnet 7.3-8, plyr 1.8.1, preprocessCore 1.28.0, proto 0.3-10, reshape2 1.4.1, rpart 4.1-8, scales 0.2.4, sendmailR 1.2-1, splines 3.1.2, stringr 0.6.2, survival 2.37-7, tools 3.1.2, xtable 1.7-4, zlibbioc 1.12.0

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